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APPLICATION OF FOURIER TRANSFORM FOR DETERMINING THERMAL METHOD FREQUENCY IN HEAVY OIL PRODUCTION

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Abstract

For wells producing heavy oil, where thermal methods are applied, it is very important to know the frequency of their application. This is especially significant in the case of cyclic application of thermal methods to maintain the planned well production. Data on the previous production history provide an excellent basis for determining the optimal frequency.

The Fourier transform enables the determination of the frequency of a particular phenomenon. By determining the absolute values of the elements of the Fourier series, the dominant frequency and lower-order harmonics that define the cyclic occurrence of a phenomenon can be identified. In practice, the procedure is reduced to calculating the amplitude of a specific phenomenon.

This paper presents a practical application of the Fourier transform for determining the frequency of performed thermal methods at oil field T.

Key words: heavy oil, thermal method, Fourier transform, frequency

Introduction. More than two-thirds of the oil produced in oil fields in Serbia is of the paraffinic type. Due to their characteristics, paraffinic oils cause certain problems during the production process. The extent and nature of these problems depend on the composition of the crude oil and the concentration of paraffin [1,2].

The dominant problem that occurs during the heavy oil production is paraffin deposition [3]. Nowadays for solving that problem, the mechanical, chemical and thermal methods are applied [4, 5]. The mechanical paraffin removal method is

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performed using a scraper moving along the wellbore and can be applied to flowing wells and gas-lift wells [6]. Chemical methods [7] include additive injection at the bottom of the well to enhance the effect of oil mixing and the beginning of activity at higher temperatures. Additives can prevent the paraffin deposition and improve flow characteristics depending on the selected additive or its combinations. The thermal methods [8] include hot oiling, downhole heaters [9], and the heating cable application [10]. Chemical and thermal methods can be applied in flowing wells as well as in wells using all types of artificial lift production methods.

In the application of thermal methods (hot oiling), it is critical to establish the optimal frequency of method repetition based on reservoir and well conditions. The analysis of a well's operational history is very important from the perspective of production management. Based on the well's operational history, an overview is obtained of production interruptions, their duration, causes, performed operations, and more. Additionally, the analysis of production history serves as a basis for planning work in wells. Production history data enable a detailed analysis of the activities performed in terms of their success and duration.

The frequency of a particular class of problems and the operations applied to resolve them, along with the fact that oil production is essentially a time series where certain signals (oil production) or phenomena (paraffin deposition) oscillate over time with characteristic frequencies and phases, allow the establishment of a fundamentally new functional relationship. For this reason, the application of the Fourier transform has been considered to determine the frequency (occurrence rate) of such events [11].

The method and methodology. Taking into account that oil and gas production data can practically be regarded as a series of discrete-time signals, the use of Fourier analysis makes it possible, through mathematical approximation of the real system, to achieve a high degree of agreement between the model and actual results.

Daily oil production data represent the amplitude of a signal that oscillates with a corresponding frequency and phase over the time period of interest for the analysis of well performance. In order to analyze changes in frequency and phase angle, daily production data must be transformed from the time domain into the frequency domain. This transformation can be carried out by applying the Laplace or Fourier transform [11, 12].

For data that oscillate with varying phases and frequencies, as is the case with daily oil and gas production data, practice has shown that the Fourier transform provides a more reliable representation; therefore, it has been applied here.

The Fourier transform of a function $F(t)$ is given by the following expression [13]:

$$F(\omega) = \int_{-\infty}^{+\infty} F(t) e^{-j2\pi\omega t}\, dt \tag{1}$$

and its inverse function

$$F(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j2\pi\omega t}\, dt. \tag{2}$$

In order for equations (1) and (2) to be applicable, the appropriate conditions ensuring the existence of the Fourier transform must be satisfied. Since daily production data are time signals with specific (characteristic) oscillations, the condition for applying the Fourier transform is fulfilled.

Additionally, the signals possess the property of causality, i.e., $F(t) = 0$ for $t < 0$, so that for practical applications the Fourier transform is reduced to the following form:

$$F(\omega) = \int_{0}^{+\infty} F(t) e^{-j2\pi\omega t}\, dt. \tag{3}$$

From this it can be seen that the Fourier transform is only a special case of the Laplace transform.

The solution of the Fourier function is a complex function consisting of a real and an imaginary part. The solution can be represented in the Cartesian coordinate system in the following form:

$$F(\varpi) = \operatorname{Re} F(\varpi) + j \operatorname{Im} F(\varpi) \tag{4}$$

and in polar form:

$$F(\varpi) = A(\varpi) e^{j\varphi(\omega)}. \tag{5}$$

The phase angle represents the phase shift of the oscillation at the output relative to the oscillation at the input and is defined by the following expression [11]:

$$\varphi = \tan^{-1}(I(\omega)/R(\omega)), \tag{6}$$

where the real and imaginary parts of the function are defined as:

$$R(\omega) = + \int_{-\infty}^{+\infty} F(t) \cos\omega t\, dt = \operatorname{Re}(F(\omega)), \tag{7}$$

$$I(\omega) = - \int_{-\infty}^{+\infty} F(t) \sin\omega t\, dt = \operatorname{Im}(F(\omega)). \tag{8}$$

The amplitude spectrum (magnitude) represents the ratio of the output to the input harmonic oscillation and is defined by the following expression:

$$A(\omega) = \sqrt{R^2(\omega) + I^2(\omega)}. \tag{9}$$

The amplitude and the phase angle can also be presented separately, where $A(\omega)$ is referred to as the amplitude-frequency characteristic and $\varphi(\omega)$ as the phase-frequency characteristic. With an increase in the frequency ω, the value of the amplitude of the output oscillation decreases; consequently, the value of the amplitude-frequency characteristic $A(\omega)$ also decreases.

The Fourier transform also enables the determination of the frequency of a particular phenomenon. By determining the absolute values of the elements of the Fourier series, the dominant frequency and lower-order harmonics that define the cyclic nature of the occurrence of a given phenomenon can be identified. In practice, the procedure is reduced to calculating the amplitude of a particular phenomenon [13].

In this way, the Fourier transform can be applied to calculate the frequency of workover operations performed in wells, acidizing treatments, thermal treatments, and similar activities. Furthermore, based on the calculated frequency of occurrence of a given phenomenon, its appearance in a future period can be predicted. This is particularly important for the periodic application of certain methods aimed at maintaining the planned well production [11].

Results and discussion. The application of the Fourier transform for determining the frequency of applying the thermal method (hot oiling) will be demonstrated using the example of oil field T.

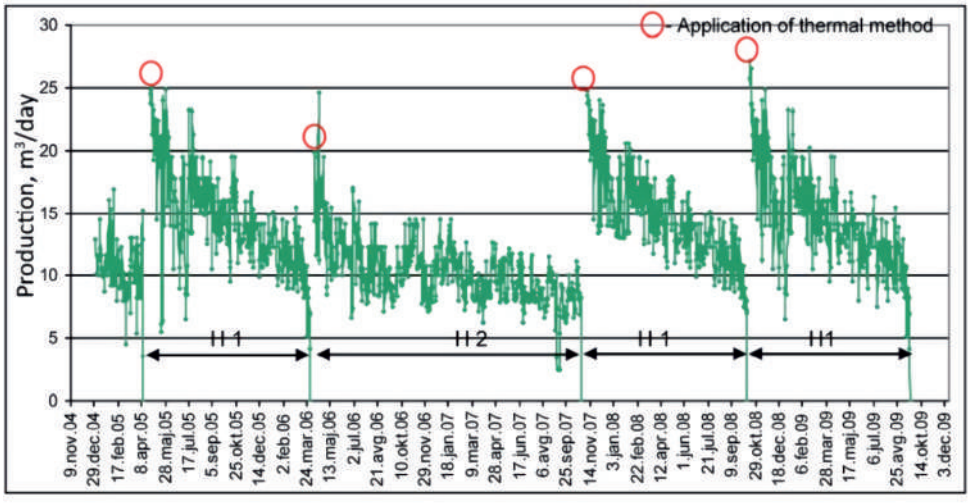
This oil field produces heavy oil from 59 wells. Sucker rod pumps were used for fluid production. The average value of pour point is around 33 °C, although for some wells reaches a value of 40 °C. The average content of paraffin is about 17%, with values in the range of 5–25%. The density of crude oil is 875 kg/m^3. The content of asphaltene is 1.16, coke 4.35, sulphur 1.01. The percentage of water content ranges from 8 to 17. Extracted crude oil according to its density belongs to medium-heavy oil, high-paraffin according to its paraffin content and high-sulphur according to its sulphur content.

The determination of the frequency of thermal treatment of wells to remove deposited paraffin will be illustrated using wells T-1 and T-2. Wells T-1 and T-2 represent typical wells with the need for cyclic application of thermal methods in order to maintain the desired oil production. Well T-1 is characterized by longer work intervals, while well T-2 by shorter.

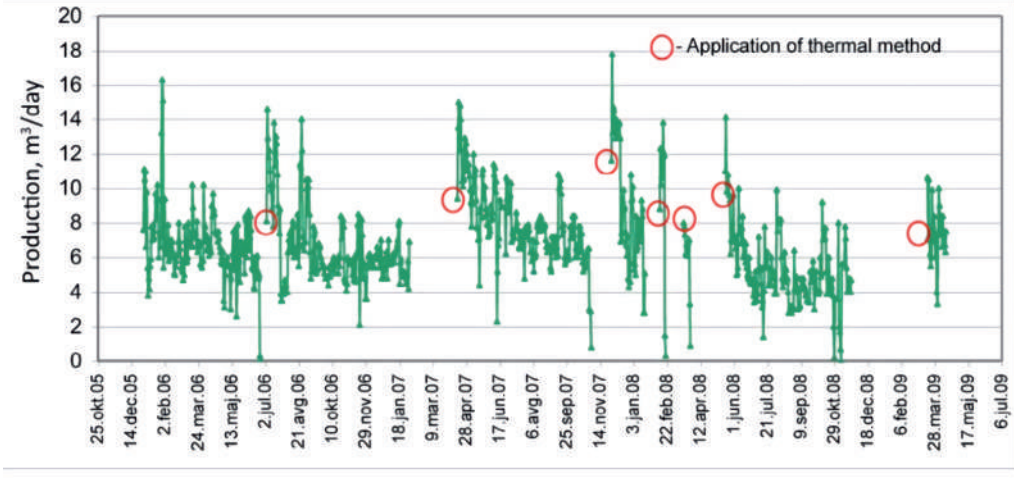
The period for development and testing lasted about 1 year. The analyzed period on well T-1 was over 5 years, while on well T-2 it was about 4.5 years.

Figure 1a shows the production of well T-1 during the analyzed period, with the times when the thermal method was applied indicated. The calculated frequencies of applying the thermal method, based on the Fourier transform, are shown in Fig. 2a.

The calculated frequencies of applying the thermal method obtained by applying the Fourier transform (Fig. 2) are 338 days for H1 and 565 days for H2 (two dominant peaks). These values show very good agreement with the actual frequencies of 337 to 349 days for H1 and 572 days for H2.



a) well T-1



b) well T-2

Fig. 1. Oil production

Figure 1b shows the production of well T2, with the times when the thermal method was applied indicated.

In well T-2, during the analyzed time period, thermal treatment of the well was performed at intervals of 8, 125, 56, and 14 days. The frequency of occurrence of the thermal treatment calculated using the Fourier transform is 10, 52, and 120 days (Fig. 2b). The calculated treatment frequencies correspond very well with the actual ones.

The analysis of the frequency of performed thermal methods, aimed at defining the optimal interval for applying thermal treatment, will be illustrated using

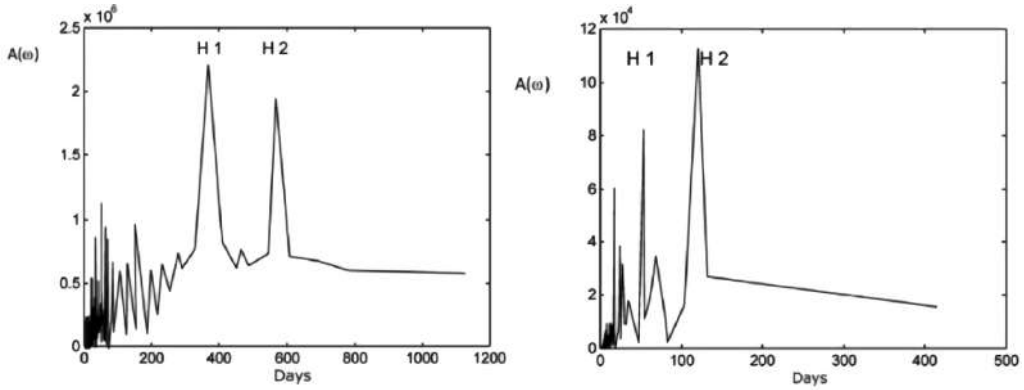


Fig. 2. Calculated frequency of application of the thermal method

wells T-3 and T-4. Table 1 shows the thermal method operations performed during the analyzed time period.

The frequencies of performing thermal methods, calculated using the Fourier transform, are shown in Fig. 3.

As seen in Fig. 3a, the dominant peak corresponds to a frequency of 16.66 days. Additionally, two other dominant peaks are observed, with frequencies of 14.20 days and 25.14 days. These two additional peaks result from the irregular intervals at which the thermal method was applied. The calculated frequency of performing the thermal method every 16.66 days fully ensures normal and continuous well operation.

In well T-4, a dominant peak is also observed, with a frequency of 19.76 days (Fig. 3b), as well as two others with frequencies of 13.14 days and 24.75 days. For

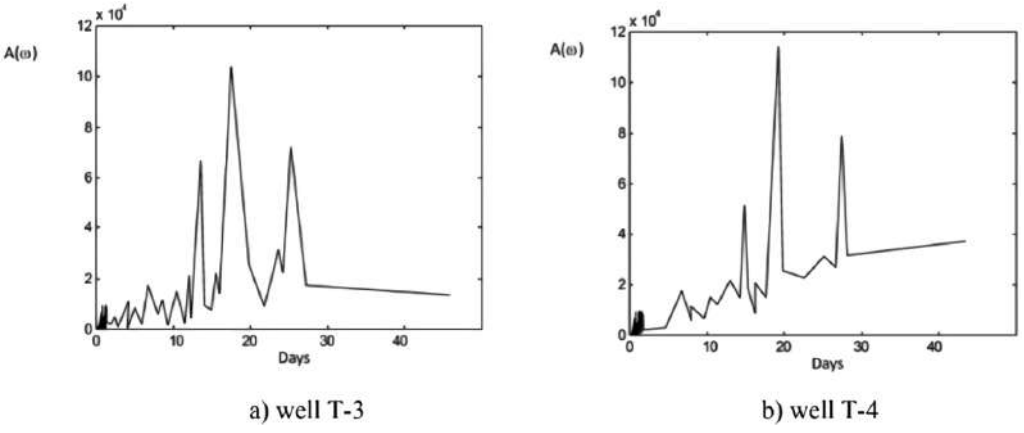


Fig. 3. Calculated frequencies of the thermal method

T a b l e 1

Application of the thermal method

Well		T-3		T-4	
Month	Year	Date of execution	Frequency	Date of execution	Frequency
August	2016	14		16	
September	2016	4, 27	24, 19	9, 28	25, 19
October	2016	14	15	13	17
November	2016	5, 30	26	9	27
December	2016	10, 19	23, 11	2, 13	23, 11
January	2017	12, 22	30, 14	13, 27	30, 14
February	2017	5, 11	11	7	11
March	2017	4, 14, 24	23, 14	2, 16	24, 14
April	2017	15, 29	14, 14	1, 15, 29	16, 14, 14
May	2017	14, 22	13, 17	12, 29	13, 17
June	2017	4, 16, 27	17, 13	12, 25	14, 13
July	2017	10, 17	15, 12	10, 22	15, 12
August	2017	6, 12, 26	17, 16	5, 21	14, 16
September	2017	8, 19, 25	16, 14	5, 19	15, 14
October	2017	4, 17, 27	17, 15, 10	6, 21, 31	17, 15, 10
November	2017	6, 7, 12, 23	11, 12	11, 23	11, 12
December	2017	2, 9, 20, 29	13, 11, 7	4, 15, 22	11, 11, 7
January	2018	3, 12, 19, 37	12, 13, 13	3, 16, 29	12, 13, 13
February	2018	4, 13, 23	11, 14	9, 26	11, 17
March	2018	5, 16, 30	13, 14	11, 25	13, 14
April	2018	7, 9	14, 19	11, 30	14, 19
May	2018	8, 27	13, 14	13, 27	13, 14
June	2018	11, 25	13	9	13
July	2018	14, 30	27, 23	6, 29	27, 21
August	2018	4, 18	20	18	20
September	2018	5, 18	27	14	27
October	2018	8, 17	25, 15	7, 22	23, 15
November	2018	17	28	20	28
December	2018	4, 16, 29	14, 25	4, 29	14, 24
January	2019	9, 21	17	16	18
February	2019	3, 16	18	3	18
March	2019	4	27, 26	2, 28	27, 26

this well, the calculated frequency of performing the thermal method every 19.76 days ensures normal and continuous well operation.

The presented examples confirm that the Fourier transform can be very successfully used as an effective tool for determining the cyclic occurrence of certain problems in well operation, which is highly important for preventive actions. This is especially significant in the case of the cyclical occurrence of specific well operation problems (such as paraffin deposition), in order to establish their occurrence period and enable preventive measures.

Conclusion. This paper presents the application of the Fourier transform for determining the frequency of performed thermal methods. Based on the production history of wells T1 and T2 the times when thermal treatment was applied were identified. By applying the Fourier transform, the frequency of the applied thermal method was calculated, showing very good agreement with the actual frequencies.

Furthermore, using wells T3 and T4 as examples, the Fourier transform was applied to determine the optimal interval for applying the thermal method. This verified the use of the Fourier transform as an effective tool for determining the cyclic occurrence of specific problems in well operation, which is extremely important for defining frequencies for preventive actions in the future.

The presented methodology also enables more efficient planning of workover operations, acidizing treatments, and other activities on wells.

REFERENCES

[1] Yarmola T., P. Topilnytskyy, V. Romanchuk (2023) High-viscosity crude oil. A review, JCCT, **17**(1), 195–202.

[2] Danilovic D. S., V. D. Karovic-Maricic, R. M. Šećerov Sokolović et al. (2011) Laboratory testing and simulation of the paraffin deposition in Turija field oil well in Vojvodina, Hemijska Industrija, **65**(3) 249–256 (in Serbian).

[3] Tinsley J. F., R. K. Prud'homme (2010) Deposition apparatus to study the effects of polymers and asphaltenes upon wax deposition, J. Petrol. Sci. Eng., **72**(1), 166–174.

[4] Crnogorac M., M. Tanasijević, D. Danilović et al. (2020) Selection of artificial lift methods: A brief review and new model based on fuzzy logic, Energies, **13**(7), 1758.

[5] Towler B. F., S. Rebbapragada (2004) Mitigation of paraffin wax deposition in cretaceous crude oils of Wyoming, J. Petrol. Sci. Eng., **45**(1), 11–19.

[6] Zhang X., J. Tian, L. Wang, Z. Zhou (2002) Wettability effect of coatings on drag reduction and paraffin deposition prevention in oil, J. Petrol. Sci. Eng., **36**, 87–95.

[7] Venkatesan R., N. Nagarajan, K. Paso et al. (2005) The strength of paraffin gels formed under static and flow conditions, Chem. Eng. Sci., **60**(13), 3587–3598.

[8] Sierra R., B. Tripathy, J. E. Bridges, S. M. Farouq (2001) Promising progress in field application of reservoir electrical heating methods, Porlamar, Margarita Island, Venezuela, March 12–14, 1–17.

[9] Roopa I., R. A. Dawe, T. Samuel (2005) Downhole heating: Productivity ratio improvements – dangers of misconceptions, Petrol. Sci. Technol., **23**, 681–692.

[10] Danilovic D., V. Karovic-Maricic, V. Cokorilo (2010) Solving paraffin deposition problem in tubing by heating cable application, Thermal Sci., **14**(1), 247–253.

[11] Danilovic D. (2001) Optimization of small oil field production using an integrated management model, University of Belgrade, Ph.D. Thesis, 161 pp.

[12] Hayatdavoudi A. (1999) Effect of water-soluble gases on production decline, production stimulation and production management, SPE 50781, 1–16.

[13] Cuomo S., G. De Pietro, R. Farina et al. (2015) A novel O(n) numerical scheme for ECG signal denoising, Procedia Comput. Sci., **51**, 775–784.

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