

MATHEMATICAL MODELLING OF MULTIPHASE INDUCTION MACHINES BASED ON THE PHASE-DOMAIN APPROACH

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Abstract

The study focuses on electromechanical energy conversion processes in induction motors with multiphase stator windings. Its aim is to develop a mathematical model using the phase-domain approach and to determine the optimal number of phases.

Mathematical modelling has progressed from phase-coordinate equations to orthogonal transformations and the vector space decomposition (VSD) method. While VSD simplifies control synthesis, phase-domain models provide better analysis of unbalanced and fault conditions.

A universal phase-domain model of a multiphase induction motor was developed, along with a method to account for changes in electromagnetic parameters when increasing the number of phases. Numerical simulations were performed in MATLAB 2024a.

Results show that increasing the number of phases raises the maximum startup torque and torque ripple. The startup time has a minimum at 5–6 phases. Considering manufacturability and inverter complexity, five-phase stator windings are recommended.

Key words: multiphase induction motor, phase-domain approach, mathematical modelling, electromagnetic torque

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Introduction. Multiphase induction motors (IMs) are machines with more than three stator phases ($m > 3$). Increasing the number of phases improves performance, although such advantages were not widely used in the 20th century due to the lack of suitable power supplies.

A higher phase number enhances the magnetic field quality by suppressing harmonics. It also enables advanced control through decomposition into independent subspaces, as seen in five-phase systems.

Another key benefit is the distribution of current across more phases, reducing current per phase and allowing the use of lower-rated inverter switches.

IMs remain the most widely used electrical machines, and their mathematical modelling is still an important and relevant research area, especially for multiphase systems.

A fundamental contribution to the analytical description of electrical machines was made by PARK [1] and CLARKE [2].

Although the concept of a multiphase drive system dates back to the mid-twentieth century, the initial pace of development was rather slow, as evidenced by the first two surveys of the field published at the beginning of this century [3, 4].

A fundamental review of the modern period is presented in [5], which is devoted to the state of the art in multiphase induction motor drive technologies. It systematizes the advantages of multiphase machines, methods of their modelling, approaches to vector control and direct torque control, as well as inverter supply topologies and PWM algorithms for five-phase and asymmetrical six-phase drives.

LEVI [6] highlights the rapid development of multiphase machine research after 2008, emphasizing the Vector Space Decomposition (VSD) method as a standard modelling tool.

In [7], a mathematical model of a wound-rotor IM is proposed in phase coordinates, preserving physical interpretability and enabling analysis of both transient and steady-state modes, with implementation in MATLAB.

In [8], a nine-phase six-terminal IM model based on VSD is developed, reducing the system to an equivalent six-phase machine with decoupled subspaces.

Literature analysis shows a clear gap: mathematical modelling of multiphase induction motors directly in the phase domain, without orthogonal transformations, remains insufficiently studied.

This approach is crucial for modelling asymmetrical windings and faults, analyzing higher-order harmonics, developing diagnostic methods based on phase variables, and systems with many phases where vector decomposition becomes inefficient.

Therefore, developing universal phase-domain models that account for arbitrary phase numbers, mutual inductances, and magnetic nonlinearities is a promising and practically important research direction.

Mathematical model of the multiphase IM. To derive the mathematical model of the multiphase IM, the theory of the generalized electromechanical converter was applied. The derivation is based on standard ideal motor assumptions:

- unsaturated magnetic circuit and uniformly distributed stator windings;
- uniformly distributed rotor squirrel-cage bars;
- replacement of distributed windings with equivalent concentrated ones having the same MMF;
- sinusoidal distribution of windings and magnetic fields in the air gap.

We will assume that the stator winding has M phases connected to the power supply. Rotor winding has N short-circuited phases, Fig 1.

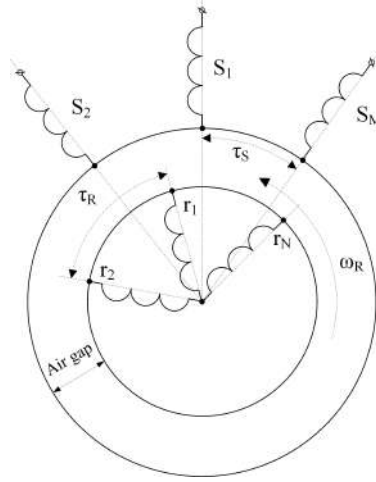


Fig. 1. Scheme of mutual arrangement of stator and rotor windings of a multiphase IM

In Fig. 1, the following notations are used: $S_1, S_2, \dots, S_m$ are the stator windings, $1 - M$; $r_1, r_2, \dots, r_n$ are the rotor windings, $1 - N$; τ_s, τ_r are the phase displacement angles between individual stator and rotor windings, respectively; ω_R is the rotor angular speed.

The system of equations of electrical equilibrium of a multiphase IM in matrix form will have a known form:

$$(1) \quad [\mathbf{U}] = [\mathbf{R}] \cdot [\mathbf{I}(t)] + \frac{d}{dt} [\mathbf{\Psi}(t)].$$

Let us introduce additional notations:

$$(2) \quad \rho_S = 2\pi/M; \quad \rho_R = 2\pi/N.$$

Vector-column of stator supply voltages has dimension $1 \times M$:

$$(3) \quad [\mathbf{U}_S] = [U_1 \ U_2 \ \dots \ U_M]^T.$$

The stator supply voltage in a general form can be written as follows:

$$(4) \quad U_k = \sqrt{2} \cdot U_f \cdot \sin(\omega_C \cdot t - \rho_S \cdot (k - 1)),$$

where $\omega_C = 2\pi \cdot f$ is the angular speed of rotation of the stator field with the frequency of the supply network f .

The supply voltage column vector for a squirrel-cage rotor is a zero vector of dimension $1 \times N$.

$$(5) \quad [\mathbf{U}_R] = [0 \quad 0 \quad \cdots \quad 0]^T.$$

Then the vector-column of supply voltages in cell-matrix form is

$$(6) \quad \mathbf{U} = \begin{bmatrix} \mathbf{U}_S \\ \cdots \\ \mathbf{U}_R \end{bmatrix},$$

and has dimension $1 \times (M + N)$.

Vector-column of stator winding currents has dimension $1 \times M$:

$$(7) \quad [\mathbf{I}_S] = [I_1 \quad I_2 \quad \cdots \quad I_M]^T.$$

Vector-column of rotor winding currents of dimension $1 \times N$:

$$(8) \quad \mathbf{I}_R = [I_{R1} \quad I_{R2} \quad \cdots \quad I_{RN}]^T.$$

Vector-column of currents in cell-matrix form is equal to

$$(9) \quad \mathbf{I} = \begin{bmatrix} \mathbf{I}_S \\ \cdots \\ \mathbf{I}_R \end{bmatrix}.$$

The square diagonal matrix of active resistances $[\mathbf{R}]$ has dimension $(M + N) \times (M + N)$. The elements of the main diagonal of this matrix are equal to the active resistances of the corresponding windings.

Vector-column of stator winding fluxes has dimension $1 \times M$:

$$(10) \quad [\mathbf{\Psi}_S] = [\Psi_1 \quad \Psi_2 \quad \cdots \quad \Psi_M]^T.$$

Vector-column of the rotor winding fluxes of dimension $1 \times N$:

$$(11) \quad \mathbf{\Psi}_R = [\Psi_{R1} \quad \Psi_{R2} \quad \cdots \quad \Psi_{RN}]^T.$$

Vector-column of fluxes in the cell-matrix form is equal to

$$(12) \quad \mathbf{\Psi} = \begin{bmatrix} \mathbf{\Psi}_S \\ \cdots \\ \mathbf{\Psi}_R \end{bmatrix}.$$

Flux equation for each winding can be written in the form:

$$(13) \quad \Psi_k = I_k \cdot L_k + \sum_{j=1, j \neq k}^{j=N} I_j \cdot M_{kj},$$

where ψ_k , I_k , L_k are the flux-coupling, current and inductance of the winding with index k ; I_j , M_{kj} current of the j -th circuit and mutual inductance of windings with indices k and j .

The stator winding eigen inductance matrix can be written in the following form:

$$(14) \quad [\mathbf{L}_{SS}] = [\mathbf{L}_{\delta S}] + [\mathbf{M}_{SS}],$$

where $\mathbf{L}_{\delta S}$ is a square diagonal matrix of leakage inductances of size $M \times M$; the value of the elements of the main diagonal is equal to the leakage inductances of the stator windings; $\mathbf{M}_{SS}$ is a square matrix of mutual inductances between the stator windings of size $M \times M$; the elements of this matrix are equal to the mutual inductances between the corresponding stator windings. Since the stator windings are stationary, the expression for the mutual inductance between the stator windings with indices j and k is:

$$(15) \quad \mathbf{M}_{SS}[j, k] = m_1 \cdot \cos(\rho_S \cdot (k - j)); j = 1 \dots M; k = 1 \dots M.$$

Similarly, taking into account the number of rotor phases, the eigen inductance matrices of the rotor windings can also be represented.

$$(16) \quad [\mathbf{L}_{RR}] = [\mathbf{L}_{\delta R}] + [\mathbf{M}_{RR}],$$

where $\mathbf{L}_{\delta R}$ is a square diagonal matrix of leakage inductances with dimension $N \times N$. The expression of the mutual inductance between the rotor windings with indices j and k can be written as follows:

$$(17) \quad \mathbf{M}_{RR}[j, k] = m_2 \cdot \cos(\rho_R \cdot (k - j)); j = 1 \dots N; k = 1 \dots N.$$

Next, a matrix of mutual inductances between the stator windings and the rotating rotor windings must be compiled.

Based on geometrical considerations, in Fig. 1, the formula for the mutual inductance between the rotor winding with number n and the stator winding with number m can be written as follows:

$$(18) \quad M_{S_m R_n} = m_0 \cdot \cos(\gamma(t) - \rho_S(m - 1) + \rho_R(n - 1)),$$

where $\gamma(t)$ is the instantaneous value of rotor rotation angle.

The matrix of mutual inductances between the stator and rotor windings will generally be a rectangular matrix of dimension $M \times N$.

Then the full matrix of mutual inductances of a multiphase IM can be represented in the following cell-matrix form:

$$(19) \quad \mathbf{M} = \begin{bmatrix} \mathbf{L}_{SS} & \vdots & \mathbf{M}_{SR} \\ \dots & \dots & \dots \\ \mathbf{M}_{SR}^T & \vdots & \mathbf{L}_{RR} \end{bmatrix}.$$

This matrix is square and has dimension $(M + N) \times (M + N)$.

The torque of an IM can be calculated by a general expression

$$(20) \quad T_E = -\frac{3}{2}Z_p \cdot \left(\vec{\Psi}_1 \times \vec{\mathbf{I}}_1 \right) = -\frac{3}{2}Z_p \cdot \left(\left| \vec{\Psi}_1 \right| \cdot \left| \vec{\mathbf{I}}_1 \right| \cdot \sin(\alpha_\Psi - \alpha_{\mathbf{I}}) \right).$$

The space vectors of stator flux-current and current can be determined from their instantaneous values, taking into account the phase shift of flux and current values of individual phases of the IM:

$$(21) \quad \vec{\mathbf{I}}_1 = \sum_{k=1}^N I_k \cdot e^{i(k-1)}.$$

The rotor motion equation based on Newton's second law:

$$(22) \quad T_E - T_L = J \frac{d\omega_r}{dt},$$

where T_E is the electromagnetic torque of IM, Nm; T_L is the load torque, Nm; J is the moment of inertia, Nm; ω_r is the rotor angular speed, s^{-1} ; t is time, s .

The above equations constitute the theoretical mathematical model of a multiphase IM.

Comparative analysis of multiphase IMs based on the mathematical model. Let us evaluate the nature and magnitude of changes in parameters for the m -phase stator winding compared to the three-phase one based on [9].

The number of turns per phase of the winding is:

$$(23) \quad W(m) = Z \cdot N_c / (2 \cdot m),$$

where Z is the number of stator slots; N_c is the number of conductors per slot.

The number of turns per phase $W(m)$ decreases inversely with the number of phases.

Let us determine the winding distribution factor k_d (m). The phase belt of one phase is given by: $\gamma = 180^\circ/m$.

The number of slots per pole per phase is determined as:

$$(24) \quad q = Z / (p \cdot m).$$

Let us denote by α the angle between adjacent slots in electrical degrees:

$$(25) \quad \alpha = 360^\circ / Z.$$

The winding factor k_W accounts for the fact that a real distributed winding produces a lower EMF than a concentrated winding with the same number of turns. For a double-layer winding with full pitch:

$$(26) \quad k_W = \frac{\sin(q \cdot \frac{\alpha}{2})}{q \cdot \sin(\frac{\alpha}{2})}.$$

The winding factor of a multiphase winding increases with the number of phases. This is due to the reduction in the phase belt width. A narrower phase belt means that the EMFs of individual conductors are less dispersed in phase, and their geometric sum approaches the arithmetic sum.

As the number of turns per phase decreases, the cross-sectional area of the turn increases. The relative increase in conductor area is:

$$(27) \quad \frac{a_m}{a_3} = \frac{W_3}{W_m} = \frac{m}{3}.$$

The active resistance of a phase of a winding is given by:

$$(28) \quad R = \rho \cdot L_W / a,$$

where the conductor length L_W is proportional to the number of turns per phase $W(m)$.

Thus, from (27), (28) we obtain the following relation:

$$(29) \quad \frac{R_m}{R_3} = \frac{W_m}{W_3} \cdot \left(\frac{a_3}{a_m} \right) = \left(\frac{3}{m} \right)^2.$$

The phase EMF $E(m)$ of the IM changes as follows:

$$(30) \quad \frac{E_m}{E_3} = \frac{W(m) \cdot k_W(m)}{W(3) \cdot k_W(3)} = \frac{3}{m} \cdot \left(\frac{k_W(m)}{k_W(3)} \right) \approx \left(\frac{3}{m} \right).$$

Taking into account (30), the magnetizing inductance L_M is related to the main air-gap flux by the expression:

$$(31) \quad \frac{L_M(m)}{L_M(3)} = \frac{m}{3} \cdot \left(\frac{E_m}{E_3} \right)^2 \approx \left(\frac{3}{m} \right).$$

The stator leakage inductance changes as follows:

$$(32) \quad \frac{L_{\sigma 1}(m)}{L_{\sigma 1}(3)} = \left(\frac{W(m)}{W(3)} \right)^2 \approx \left(\frac{3}{m} \right)^2.$$

The stator leakage inductance decreases quadratically, similarly to the stator phase resistance. Using expressions (29), (31), (32), we will determine the parameters of the IM when switching to a multiphase design.

For mathematical modelling of multiphase IM, motor AXR 500MP4 with the rated power 1500 kW, rated voltage 10 000 V, rated speed 984 rpm was used.

The solution of the proposed equations was implemented by means of a program in MATLAB 2004a.

The obtained results are presented in Fig. 2 and Fig. 3.

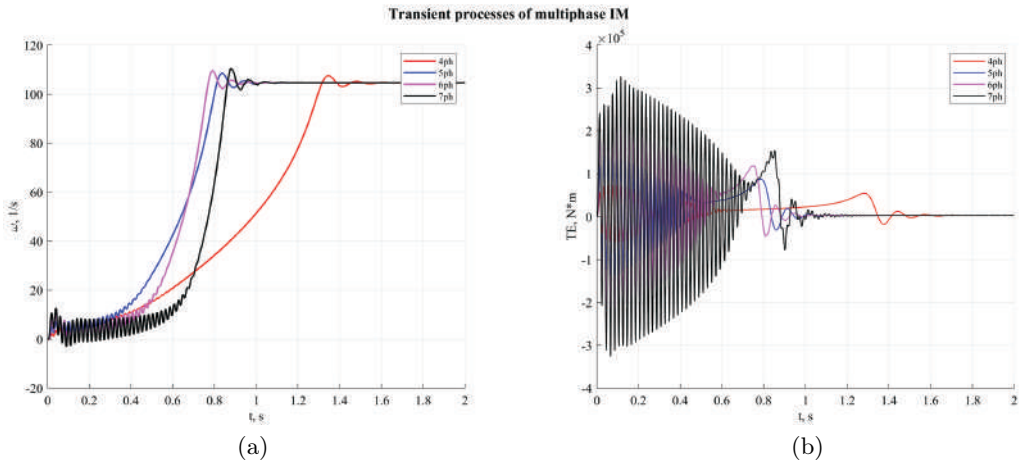


Fig. 2. Transient characteristics of the direct start of a multiphase IM: (a) rotor angular speed; (b) electromagnetic torque

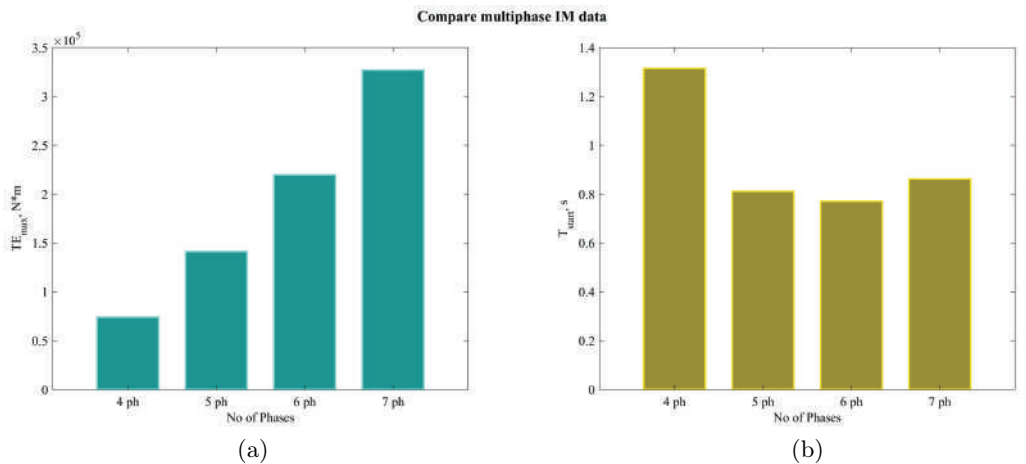


Fig. 3. Diagrams of transient performance indicators of the start-up of a multiphase induction motor: (a) maximal electromagnetic torque; (b) duration

Figure 2 shows the diagrams of direct starting of multiphase induction motors with different (4 to 7) numbers of phases.

Figure 3 shows diagrams of the maximal torque of IM during the start-up process and the duration of the start-up process.

An increase in the number of phases leads to a higher maximum electromagnetic torque of the IM during the starting process. At the same time, the amplitude of the variable component of the electromagnetic moment will increase.

The dependence of the starting time on the number of phases exhibits a pronounced minimum at $m = 5 - 6$. A further increase in the number of stator phases leads to an increase in the starting time.

Conclusion. The study confirms the practical significance of developing phase-domain models of multiphase IMs. A method for recalculating electromagnetic parameters of multiphase winding and a model of multiphase IM are proposed.

Increasing the number of phases increases the starting torque, but also increases torque pulsation. Starting time is minimal at $m = 5 - 6$, whilst a further increase in the number of phases results in a longer starting time.

The improved performance is due to a reduction in the phase belt width. The increase in starting time with a higher number of phases is due to torque pulsations. The results confirm the feasibility of using multiphase asynchronous motors in traction drives.

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