Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences
Tome 79, No 11, 2026

ENGINEERING SCIENCES
Digital signal processing

GRAPHICAL-ANALYTICAL METHOD FOR DETERMINING ACTIVE POWER LOSSES IN POWER SUPPLY SYSTEMS

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Received on June 8, 2026
Presented by Ch. Roumenin, Member of BAS, on September 29, 2026

Abstract

The current study proposes a graphical representation of an analytical method for determining active power losses in power systems depending on various energy parameters. In graphical form, partial losses are presented as a function of $\cos\varphi$, as well as various ratios between $\cos\varphi$ and the power factor PF.

Key words: active power losses, power systems, power factor, power quality, non-sinusoidal conditions, load asymmetry, load unbalance, harmonic distortion, IEEE Std 1459, reactive power

Introduction. Power and electrical energy losses can be classified into two main groups – conventional and operational. The first group of losses depends on the parameters and characteristics of the components and electrical equipment, as well as on the power supply system as a whole. These are, generally, the resistances and, respectively, the conductivities of components of an active, inductive and capacitive nature, as well as certain parameters of the electromagnetic field, amongst others. During the operation of the power supply system, these losses manifest as relatively steady values which remain almost constant over time and may be regarded as constant. The second group of losses depends on the operating conditions of the equipment, primarily on fluctuations in voltage and current,

The authors gratefully acknowledge the financial support from the Bulgarian National Science Fund (grant agreement KP-06-COST12/13.08.2025), COST Action CA21104: Pan-European Network for sustainable Hydropower (PEN@Hydropower).
https://doi.org/10.7546/CRABS.2026.11.12

on the non-linear nature of the components, and on the operating modes of the loads. These vary constantly over time, and both deterministic and probabilistic-statistical approaches are used to determine them. The dynamics of the load and changes in the voltages and currents of the three phases determine higher harmonics, which create additional substances of the power and electrical energy losses. Furthermore, the stochastic nature of the load creates a dispersion component of the power, which gives rise to an additional component of the active losses. The precise determination of all these loss components depends to a large extent on the accuracy of the calculations of the various types of power.

Exposition. The analytical approach presented in [1] allows to determine the partial power losses using the expressions:

$$
\begin{aligned}
&\text{Partial reactive power losses } \Delta P_Q = \frac{Q^2}{P^2}.100[\%] \\
&\text{Partial losses from pulsating power } \Delta P_N = \frac{N^2}{P^2}.100[\%] \\
&\text{Partial latent power losses } \Delta P_G = \frac{G^2}{P^2}.100[\%] \\
&\text{Partial deformation power losses } \Delta P_D = \frac{D^2}{P^2}.100[\%]
\end{aligned} \tag{1}
$$

where P, Q and S are the active, reactive and apparent power in symmetrical and sinusoidal mode, determined by the ratios $S = \sqrt{P^2 + Q^2}$; $Q = P.\operatorname{tg}\varphi$; φ is the phase shift angle between the active and apparent power in the power triangle; N is the pulsating power caused by current asymmetry (negative sequence components); G is the latent power caused by current unbalance (zero sequence components); D is the distortion power caused by higher harmonics of the currents [2, 3, 5].

Based on the power definitions established in IEEE Std 1459 [1], the reactive power Q and the inactive powers N, G, and D can be mathematically expressed through the respective unbalance and non-sinusoidality coefficients as follows:

$$
Q = P.\operatorname{tg}\varphi; \qquad N = \varepsilon_I.S; \qquad G = \alpha_I.S; \qquad D = K_i.S, \tag{2}
$$

where ε_I and α_I are the three-phase asymmetry and unbalance coefficients, determined by the expressions $\varepsilon_I = I_2/I_1$; $\alpha_I = I_0/I_1$; I_1, I_2, I_0 are the positive, negative and zero sequence currents obtained after applying the Fortescue method; $I_1 = \frac{1}{3}\left(I_A + aI_B + a^2 I_c\right)$; $I_2 = \frac{1}{3}\left(I_A + a^2 I_B + aI_C\right)$; $I_0 = \frac{1}{3}\left(I_A + I_B + I_C\right)$, a is the Steinmetz operator; $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$; $1 + a + a^2 = 0$; $a^3 = 1$; $a^4 = a$; K_i is the

three-phase non-sinusoidality coefficient, determined by the expression:

$$K_i = \left[\frac{\sum\limits_{\nu=2}^{\infty} I_{A_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{B_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{C_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{n_\nu}^2}{I_{A_1}^2 + I_{B_1}^2 + I_{C_1}^2 + I_{n_1}^2 + \sum\limits_{\nu=2}^{\infty} I_{A_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{B_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{C_\nu}^2 + \sum\limits_{\nu=2}^{\infty} I_{n_\nu}^2} \right]^{0.5},$$

where I_{A_1}, I_{B_1}, I_{C_1}, I_{n_1}, are the effective values of the currents of the three phases and the zero at fundamental frequency; I_{A_ν}, I_{B_ν}, I_{C_ν} and I_{n_ν}, are the effective values of the harmonic currents of the three phases and the neutral.

Substituting expressions (2) into expression (1), the following expressions are obtained for the partial power losses [2, 4]:

$$\Delta P_Q = \mathrm{tg}^2\,\varphi.100\% \tag{3}$$

$$\Delta P_N = \frac{1}{\cos^2\varphi}\varepsilon_I^2.100\% \tag{4}$$

$$\Delta P_G = \frac{3n+1}{\cos^2\varphi}.\alpha_I^2.100\% \tag{5}$$

$$\Delta P_D = \frac{1}{\cos^2\varphi}(1-\varepsilon^2)\frac{K_i^2}{1-K_i^2}.100\%, \tag{6}$$

where n is the coefficient indicating the multiplicity of the resistances of the neutral conductor and phase conductors; $\varepsilon = \sqrt{\varepsilon_I^2 + (3n+1)\,.\alpha_I^2}$ is the generalized asymmetry and unbalance coefficient.

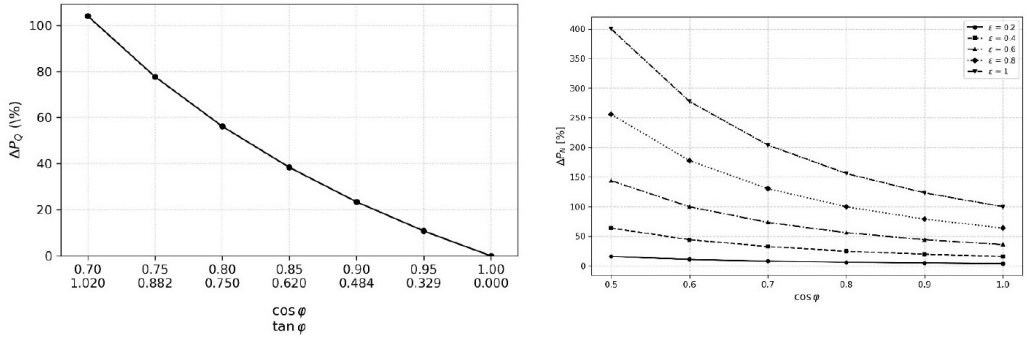

Fig. 1. Relationship $\Delta P_Q = f(\mathrm{tg}\,\varphi)$; $\Delta P_N = f(\cos\varphi.\varepsilon_i)$

Figure 1 presents the relationships $\Delta P_Q = f(\mathrm{tg}\,\varphi)$; $\delta P_N = f(\cos\varphi)$ at a given parameter ε_I. Figure 2 presents the relationships $\Delta P_G = f(\cos\varphi)$ at a given parameter α_I and $n = 1$; $\Delta P_D = f(\cos\varphi)$ at a given parameter ε and K_i, plotted in accordance with expressions (3) to (6).

The power factor (PF) represents the ratio of the active power P to the total apparent power S_D, which accounts for the asymmetry and non-sinusoidal nature

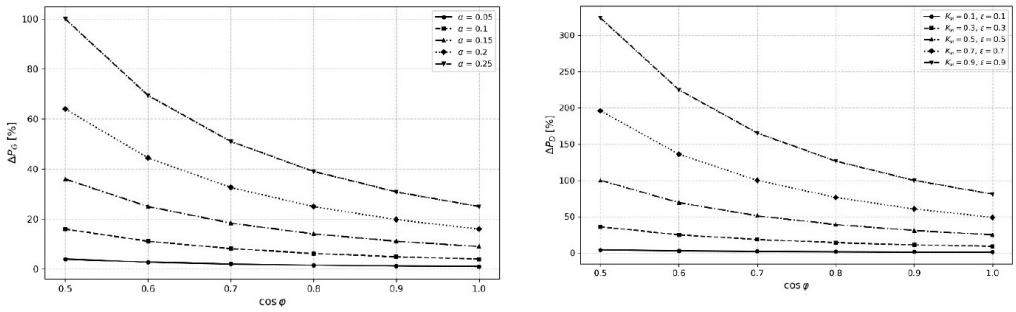

Fig. 2. Relationship $\Delta P_G = f(\cos\varphi.\alpha_I)$; $\Delta P_D = f(\cos\varphi, \varepsilon, K_i)$

of the power consumption, respectively, the presence of N, G, and D:

$$PF = \frac{P}{S_D} = \frac{P}{\sqrt{S^2 + N^2 + (3n+1)\,.G^2 + D^2}} = \frac{P}{\sqrt{P^2 + Q^2 + N^2 + (3n+1)\,.G^2 + D^2}}. \tag{7}$$

The conventional method, applied until recently in power engineering using only $\cos\varphi$, can be presented in a formalized manner using the formula:

$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}. \tag{8}$$

When comparing (7) and (8), the following inequality is obtained:

$$PF < \cos\varphi. \tag{9}$$

The traditional approach to analyzing power consumption based on $\cos\varphi$ does not allow for a precise assessment of the actual energy processes within the electric power system. The use of the conventional $\cos\varphi$ is permissible only under an approximately uniform load profile and a relatively symmetrical and sinusoidal operating regime. Otherwise, a more adequate and reliable approach is to use the power factor (PF), which accounts for both the dynamics of consumption and its distortion processes.

By accounting for the representation of Q, N, G, and D from expressions (2), after rearranging (7), the expression for PF can be written as [1]:

$$PF = \cos\varphi.\sqrt{\frac{1 - K_i^2}{1 + \varepsilon_I^2 + (3n+1)\alpha_I^2}} = \cos\varphi\sqrt{\frac{1 - K_i^2}{1 + \varepsilon^2}}. \tag{10}$$

The relationships $PF = f(\cos\varphi)$ are presented in Fig. 2 with parameters: K_i and ε, which take the pairs of values ($K_i = 0.1$, $\varepsilon = 0.1$); ($K_i = 0.3$, $\varepsilon = 0.3$); ($K_i = 0.5$, $\varepsilon = 0.5$); ($K_i = 0.7$, $\varepsilon = 0.7$); ($K_i = 0.9$, $\varepsilon = 0.9$).

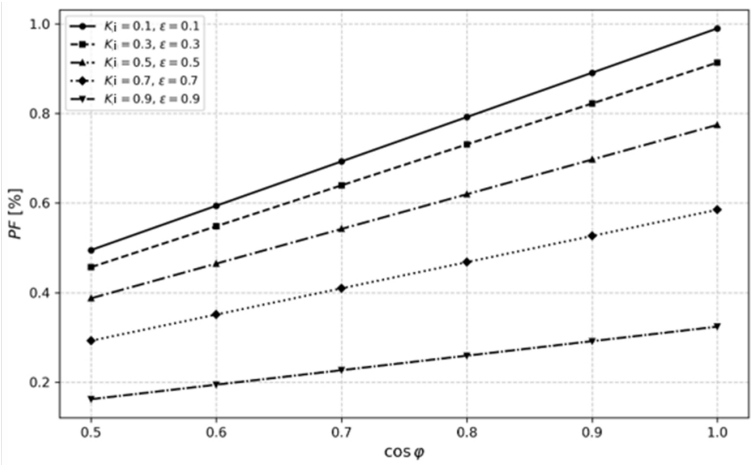

Fig. 3. Relationship $PF = f(\cos\varphi, \varepsilon, K_i)$

Under asymmetrical, unbalanced, and non-sinusoidal currents, the active power losses ΔP in the system elements and during the transfer of active power P are determined by the relationship:

$$\Delta P \approx \frac{1}{PF} = \frac{1}{P^2}.\left[P^2 + Q^2 + N^2 + (3n+1).G^2 + D^2\right]. \tag{11}$$

Under a fully symmetrical, balanced, and sinusoidal regime and without reactive power transfer (i.e. Q, N, G, and D are equal to zero), expression (11) takes the form:

$$\Delta P_{\min} = \frac{P^2}{P^2}.100\% = 100\%. \tag{12}$$

Consequently, the power components Q, N, G, and D determine the additional active power losses exceeding the baseline 100%. In accordance with (11), the percentage contribution of each power component to the total active power losses ΔP can be determined, where each is represented as a partial component according to expression (1).

The determination of ΔP using the method that was, until recently, generally accepted and which utilises $\cos\varphi$, is carried out using the following expression:

$$\Delta P \approx \frac{1}{\cos^2\varphi} \tag{13}$$

The percentage increase in losses $\delta(\Delta P)$ when using partial technology can be evaluated using the expression:

$$\delta(\Delta P) = \left[\left(\frac{\cos\varphi}{PF}\right)^2 - 1\right].100\% = \left[\left(\frac{S_d}{S}\right)^2 - 1\right].100\%. \tag{14}$$

Figure 4 shows the relationship $\delta(\Delta P) = f(\cos\varphi/PF)$ as a percentage change.

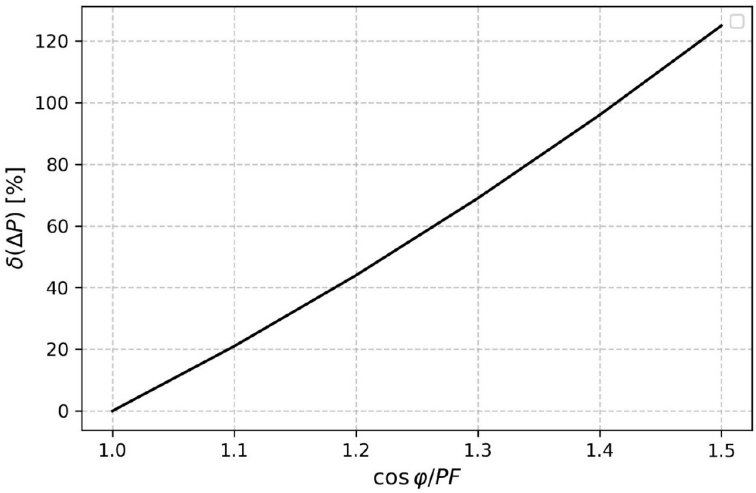

Fig. 4. Relationship $\delta(\Delta P) = f(\cos\varphi/PF)$

Conclusion. The proposed methodology for evaluating the influence of asymmetrical and non-sinusoidal operating regimes on power losses is illustrated graphically in Fig. 3. It presents the relationship between the ratio $\cos\varphi/PF$ and the relative variation of active power losses, $\delta(\Delta P)$. It can be observed that as the ratio $\cos\varphi/PF$ increases, the relative deviation of active power losses increases significantly. In particular, when $\cos\varphi/PF = 1.1$, the increase in losses is approximately 20%, while at $\cos\varphi/PF = 1.5$, the deviation exceeds 120%. This demonstrates that the use of $\cos\varphi$ instead of the true power factor PF can lead to a substantial misestimation of the power characteristics and associated losses under non-sinusoidal and asymmetrical operating conditions. Therefore, PF provides a more appropriate indicator for assessing the actual power-transfer conditions in electric power systems with distorted and unbalanced currents.

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