Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences
Tome 75, No 12, 2022

ENGINEERING SCIENCES
Mechanics

DETERMINATION OF THE SAFETY ZONES ACCORDING TO THE BEARING CAPACITY OF OVERLOADED STRUCTURAL ELEMENTS

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Received on July 11, 2022
Accepted on October 25, 2022

Abstract

The study presents a Methodology of finding the Safety zones of structural elements undergoing extreme overloading, whereas the structure retains its bearing capacity. The methodology is applied to a set of diffusively strengthened concrete beam elements. Two cases of element deformation are considered – Case 1 (elastic deformation), and Case 2 (elastic deformation accompanied by compression micro-damage). Safety zones are outlined within a Space of events introduced by the authors. Their areas have different dimensions in Case 1 and Case 2, characterizing the degree of structure safety under overloading.

Key words: design loading, overloading, safety zones, reinforced structural elements

1. Introduction. We present a Methodology for finding the Safety zones of certain structural elements undergoing extreme external overloading. However, the structure retains its load-bearing capacity.

As for the overloading, we consider all loads which exceed the admissible ones as specified by the structural norms. These can be hurricanes, powerful earthquakes and other repeating impacts, whose probability can be assessed by experts.

The Methodology operates over a group of diffusively strengthened concrete beam elements. The strengthening material diffusively penetrates the element

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DOI:10.7546/CRABS.2022.12.14

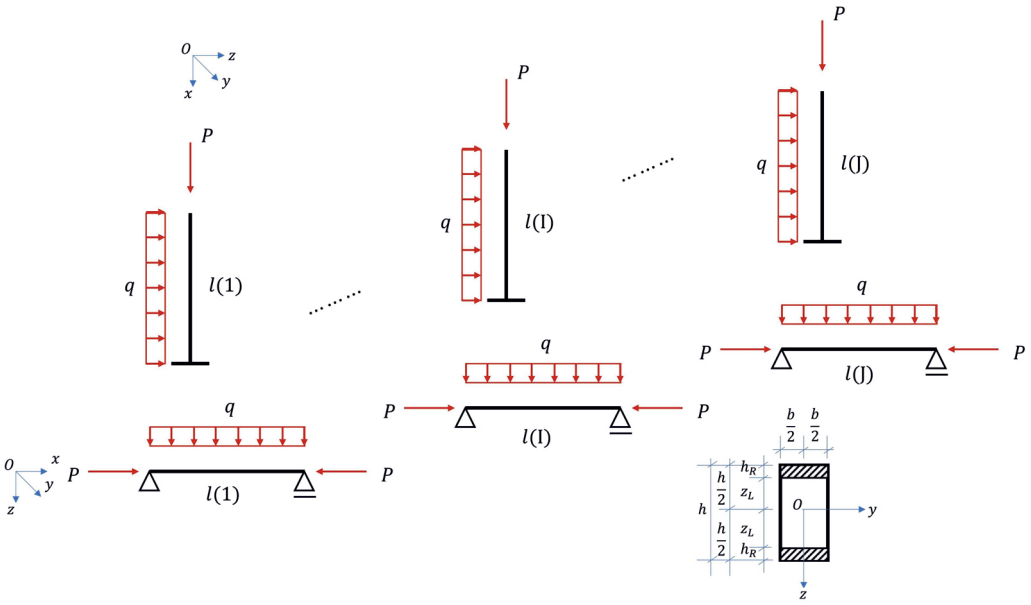


Fig. 1. A group of structural elements and a cross section consisting of a concrete core and two transition zones

forming a so-called transition layer with mechanical characteristics better than those of concrete. We find layer's geometry and describe the change of material mechanical properties throughout the layer applying an approximate computational model (ACM) [1]. We further develop the ACM and employ it to find the bearing capacity of simple and cantilever beams with a rectangular cross section undergoing extreme bending from a uniformly distributed load q and axial compression from load P (Fig. 1).

To find the Safety zones, we introduce a Space of events specified by: (i) element geometry; (ii) concrete physical-mechanical properties; (iii) applied external loading. Consider two cases of element deformation – Case 1 (elastic deformation), and Case 2 (elastic deformation accompanied by compression micro-damage). Introduce criteria of loss of element's bearing capacity – an exact criterion for simple beams and an approximate one for cantilever beams. Note that we introduce an approximate criterion to perform a quicker assessment of the element bearing capacity during the practical application of the proposed ACM [2].

2. Basic assumptions. ***2.1. Loading.*** We express overloading P and Q via normative loads P_0 and Q_0 ($P > P_0 > 0$ and $Q > Q_0 > 0$) and consider the probabilities of overloading occurrence, υ_p and υ_q, respectively. Loading is simple, it depends on a parameter ξ and one can discretely express it as

$$P(I) = P_0(I)\upsilon_p\xi, \quad Q(I) = Q_0(I)\upsilon_q\xi, \quad I = 1, 2, \ldots, J, \tag{1}$$

$$P_0(I) = k_p(I)\widetilde{P}_0(I), \quad Q_0(I) = k_q(I)\widetilde{Q}_0(I), \tag{2}$$

where $\widetilde{Q}_0(I)$ and $\widetilde{P}_0(I)$ are the basic values of overloading for a certain structural element while $k_q(I)$ and $k_p(I)$ are ratio coefficients.

Note. All parameters denoted by a tilde are related to the basic values.

Since loading is simple, we can apply the principle of superposition involving force P and maximal moment $M = Ql_r$, where l_r is a reduced length depending on the boundary conditions (Fig. 1)

$$\varepsilon_x = \varepsilon_x(M) + \varepsilon_x(N^-), \quad \sigma_x = \sigma_x(M) + \sigma_x(N^-). \tag{3}$$

2.2. Homogenization. We assess the bearing capacity and find the safety zones of homogenized structural elements with respect to their physical-mechanical characteristics affected by material strengthening. Hence, we introduce a generalized bending stiffness and generalized stiffness due to the axial load [1]

$$K^*_{\mathrm{II}} = E_B J^*_y, \quad K^*_{\mathrm{I}} = E_B F^*, \tag{4}$$

where E_B is the elastic modulus of concrete, J^*_y is the generalized inertial moment, and F^* is the generalized area of the cross section.

Note. All parameters denoted by an asterisk are related to generalized quantities, i.e., to the homogenized element.

2.3. Bearing capacity criteria. There are two criteria concerning Case 1 (C1) – Criterion 1 (KI) for tension and Criterion 2 (KII) for compression. Assume that the weaker material, i.e., concrete in this case, is the first material to lose its bearing capacity. Then, according to KI, the maximal stress σ^+_x within the cross section should not exceed the elasticity limit of concrete in tension σ^+_{BP} divided by the safety coefficient in tension C^+. Next, according to KII, σ^-_x should not exceed the elasticity limit under compression σ^-_{BP} divided by the general safety coefficient under compression C^- [3]

$$\sigma^+_{B\,per} = \max \sigma^+_x = \frac{\sigma^+_{BP}}{C^+}, \quad \sigma^-_{B\,per} = \max \sigma^-_x = \frac{\sigma^-_{BP}}{C^-}. \tag{5}$$

Considering C2 we have again two criteria – (KI) for tension and (KIII) for compression. KI is identical to that in Case 1. Under compression, we consider micro-damage and according to KIII the maximal stress σ^-_x should not exceed the limit strength of concrete σ^-_{BC} divided by the general safety coefficient under compression C^-

$$\sigma^+_{B\,per} = \max \sigma^+_x = \frac{\sigma^+_{BP}}{C^+}, \quad \overline{\sigma}^-_{B\,per} = \max \sigma^-_x = \frac{\sigma^-_{BC}}{C^-}. \tag{6}$$

2.4. Stresses and strains. Strains within the concrete core are small according to the linear strain theory, following the geometry hypotheses for plane cross sections under simple bending and compression [4]. Quadratic distribution

of the elasticity moduli and normal strains according to the extended ACM is assumed [2]. Note in C1 that the stress–strain relation under tension and compression follows Hooke's law. As for C2, the relation under tension is identical to that in C1, while it is nonlinear under compression. It is assumed to be [3,5]

$$\sigma_x^- = \overline{\sigma}_{B\,per}^- R\left(\left|\overline{\varepsilon}_x^-\right|\right), \tag{7}$$

where $R\left(|\overline{\varepsilon}_x^-|\right) = \dfrac{k_B\left|\overline{\varepsilon}_x^-\right| - \left(\overline{\varepsilon}_x^-\right)^2}{1-(k_B-2)\left|\overline{\varepsilon}_x^-\right|}$, $\overline{\varepsilon}_x^- = \dfrac{\varepsilon_x^-}{\varepsilon_{BP}^-}$, and k_B is a normative coefficient.

3. Introduction of Space of events. To outline the Safety zone we introduce the so-called Space of events specified by three "events": (i) geometry of the structural element – η; (ii) concrete physical-mechanical properties – e; (iii) applied external load – ξ.

3.1. Events presentation in a non-dimensional form. We present the three events in a non-dimensional form for convenience:

(i) *Element geometry*. Its discrete form is as follows (Fig. 1):

$$\eta(I) = \frac{b(I)}{\widetilde{b}}\frac{F^*(I)}{J_y^*(I)} l_r(I)|z_L(I)|, \quad I = 1, 2, \ldots, J \tag{8}$$

and the approximation of eq. (8) yields

$$\eta = \frac{b}{\widetilde{b}}\frac{F^*}{J_y^*} l_r|z_L|, \tag{9}$$

where l_r is the element reduced length (Fig. 1).

(ii) *Concrete mechanical properties*. The second event is presented by the deformation e, which is non-dimensional.
It is denoted by e_1 in C1 and takes the form

$$e_{\mathrm{I,II}} = |\varepsilon_{B\,per}^{\pm}| = \frac{|\sigma_{B\,per}^{\pm}|}{E_B}. \tag{10}$$

considering criteria (5) for (KI) and (KII). The deformation expresses the limit strains in tension and under compression without micro-damage.
Denote deformation by e_2 in case C2. Considering criteria (KI) and (KIII) – eq. (6), e_2 takes the form

$$e_{\mathrm{I}} = \varepsilon_{B\,per}^{+} = \frac{\sigma_{B\,per}^{+}}{E_B}, \quad e_{\mathrm{III}} = |\overline{\varepsilon}_{B\,per}^-| = \frac{|\overline{\sigma}_{B\,per}^-|}{E_B}. \tag{11}$$

Here, the deformation expresses the limit strains in tension and under compression accompanied by micro-damage.
If we select a specific concrete brand, deformation becomes constant, and the space of events becomes two-dimensional.

(iii) *Applied external load.* The applied loading is the third event. It is presented by eq. (1) in an approximate form for cases C1 and C2, using a loading parameter

$$\xi_{1,2} = \frac{P}{P_0 \upsilon_p} = \frac{Q}{Q_0 \upsilon_q}. \tag{12}$$

Hence, the space of events takes the form $\Pi_i = \{\eta_i, e_i, \xi_i\}$, $i = 1, 2$, for both cases C1 and C2, reading $\Pi_i = \{\eta_i, \xi_i\}$, $i = 1, 2$, in the special two-dimensional case.

4. Determination of the Safety zones. We present a methodology of determining the safety zones in a 2D space of events $\Pi_i = \{\eta_i, \xi_i\}$, $i = 1, 2$, for C1 and C2.

4.1. Determination of the limit curves. We find the limit curves in the first quadrants of Π_1 and Π_2 corresponding to the assumed criteria, eq. (5) and eq. (6). Note that event ξ in both cases, including criteria (C1, KI and KII, and C2, KI and KIII), reads $\xi_{1\mathrm{I}} = \xi_{2\mathrm{I}} = \xi_{\mathrm{I}}$, $\xi_{1\mathrm{II}} = \xi_{\mathrm{II}}$, $\xi_{2\mathrm{III}} = \xi_{\mathrm{III}}$.

Hence, we should design three limit curves $\xi_{\mathrm{I}} = f_{\mathrm{I}}(\eta_1)$, $\xi_{\mathrm{II}} = f_{\mathrm{II}}(\eta_1)$ and $\xi_{\mathrm{III}} = f_{\mathrm{III}}(\eta_2)$.

4.1.1. Determination of the limit curves in tension for C1 and C2. The limit curve in tension is $\xi_{\mathrm{I}} = f_{\mathrm{I}}(\eta_1)$.

The limit admissible strain in tension for KI and for a simple and cantilever beam reads in both cases as:

$$e_{\mathrm{I}} = \frac{\sigma^{+}_{B\,per}}{E_B} = \left(\frac{Q l_r}{K^*_{\mathrm{II}}} |z_L| - \frac{P}{K^*_{\mathrm{I}}} \right), \tag{13}$$

where the moment is maximal at $z = z_L$ (Fig. 1).

Relation (13) is found from eq. (3), considering criteria (5).

We substitute the general stiffnesses in eq. (13) with their values specified by eq. (4). We find the following relation for the admissible stress in tension for KI in both cases, and for a simple and a cantilever beam

$$\sigma^{+}_{B\,per} = \frac{Q l_r}{J^*_y} |z_L| - \frac{P}{F^*}. \tag{14}$$

We also express ratio coefficients k_p and k_q introduced in eq. (2) as $k_p = \dfrac{F^*}{\widetilde{F}^*}$ and $k_q = \dfrac{b}{\widetilde{b}} k_p = \dfrac{b}{\widetilde{b}} \dfrac{F^*}{\widetilde{F}^*}$. They express here the distributed normative loads. Hence, we obtain the following external overloads P and Q from eq. (1) and eq. (2)

$$P = \frac{F^*}{\widetilde{F}^*} \widetilde{P}_0 \upsilon_p \xi_{\mathrm{I}}, \quad Q = \frac{b}{\widetilde{b}} \frac{F^*}{\widetilde{F}^*} |\widetilde{Q}_0| \upsilon_q \xi_{\mathrm{I}}. \tag{15}$$

Substituting eq. (4) and eq. (14) in eq. (13), we find:

$$E_B e_{\mathrm{I}} = \begin{pmatrix} \dfrac{\overset{\approx}{b}}{\overset{\approx}{b}} \dfrac{F^*}{\widetilde{F}^*} l_r |z_L| \, |\widetilde{Q}_0| \upsilon_q \xi_{\mathrm{I}} & \dfrac{F^*}{\widetilde{F}^*} \widetilde{P}_0 \upsilon_p \xi_{\mathrm{I}} \\ \dfrac{\overset{\approx}{b}}{J_y^*} \dfrac{F^*}{F^*} & \end{pmatrix}.$$

Substituting $\eta_1 = \dfrac{\overset{\approx}{b}}{\overset{\approx}{b}} \dfrac{F^*}{J_y^*} l_r |z_L|$, $C_{\mathrm{I}} = \dfrac{\sigma^+_{B\,per} \widetilde{F}^*}{|\widetilde{Q}_0| \upsilon_q}$ and $A = \dfrac{\upsilon_p}{\upsilon_q} \dfrac{\widetilde{P}_0}{|\widetilde{Q}_0|}$ we find the equation of the limit curve in tension

$$\xi_{\mathrm{I}} = \frac{C_{\mathrm{I}}}{\eta_1 - A}. \tag{16}$$

4.1.2. Determination of the limit curve under compression for C1. The limit curve under compression has the form $\xi_{\mathrm{II}} = f_{\mathrm{II}}(\eta_1)$ for C1.

Following the above steps, we find

$$\xi_{\mathrm{II}} = \frac{C_{\mathrm{II}}}{\eta_1 + A}, \quad C_{\mathrm{II}} = \frac{|\sigma^-_{B\,per}| \widetilde{F}^*}{|\widetilde{Q}_0| \upsilon_q}. \tag{17}$$

The cross point between the two curves ξ_{I} and ξ_{II} is at η_1^*, and it is found for $\xi_{\mathrm{I}} = \xi_{\mathrm{II}}$.

4.1.3. Determination of the limit curve under compression for C2. The limit curve under compression for C2 reads $\xi_{\mathrm{III}} = f_{\mathrm{III}}(\eta_2)$.

Following the above steps, we find

$$\xi_{\mathrm{III}} = \frac{C_{\mathrm{III}}}{\eta_2 + A}, \quad C_{\mathrm{III}} = \frac{|\overline{\sigma}^-_{B\,per}| \widetilde{F}^*}{|\widetilde{Q}_0| \upsilon_q}. \tag{18}$$

The cross point between the two curves ξ_{I} and ξ_{III} is at η_2^*, and it is found for $\xi_{\mathrm{I}} = \xi_{\mathrm{III}}$.

Hence, we obtain two summary limit curves: for C1, $\xi_1 = \min\{\xi_{\mathrm{I}} \times \xi_{\mathrm{II}}\}$ and for C2, $\xi_2 = \min\{\xi_{\mathrm{I}} \times \xi_{\mathrm{III}}\}$, accounting for specific probabilities υ_p and υ_q, and considering only the first quadrant of the real positive values in coordinate systems $O\eta_1\xi_1$ for C1 and $O\eta_2\xi_2$ for C2.

The Safety zone sought for C1 is the area S_1 under the summary curve ξ_1, and the area S_2 under the summary curve ξ_2 for C2. The efficiency of the record of compression micro-damage is expressed by the enlarged area $S_2 > S_1$.

5. Example. We perform a numerical example to test the designed methodology.

Geometry characteristics. Consider both types of beams (Fig. 1). Select six cross sections $(b \times h)$ cm: (20×30), (25×35), (30×40), (35×45), (40×50), (45×55) of beams with a reduced length $l_r = 100$ cm. Assume the smallest cross

section to be the basic one. The other geometry parameters are $z_L = 14$ cm, $h_R = 1$ cm.

Material characteristics. $E_B = 3.5 \cdot 10^6$ N/cm^2, $E_R = 30 \cdot 10^6$ N/cm^2, $\sigma^+_{B\,per} = 300$ N/cm^2, $|\sigma^-_{B\,per}| = 1031$ N/cm^2, $|\overline{\sigma}^-_{B\,per}| = 1200$ N/cm^2, $K^*_{\rm I} = 2.062 \cdot 10^9$ N, $K^*_{\rm II} = 16.4 \cdot 10^{10}$ N/cm^3.

External loading. The basic values of overloading are $\widetilde{Q}_0 = 2000$ N, $\widetilde{P}_0 = 100$ N.

We find the Safety zone, executing the following calculation steps:

1. Assume some arbitrary values of probabilities υ_p and υ_q.

2. Determine the limit curves $\xi_{\rm I}$, $\xi_{\rm II}$ and $\xi_{\rm III}$ using expressions (16), (17) and (18).

3. Find the cross point between curves $\xi_{\rm I}$ and $\xi_{\rm II}$, and curves $\xi_{\rm I}$ and $\xi_{\rm III}$, i.e. find points η^*_1 and η^*_2.

   Note. If curves do not cross each other, vary the probability values. Curves cross each other for probabilities $\upsilon_p = 0.80$ and $\upsilon_q = 0.01$, and the cross points are $\eta^*_1 = 7.3$ and $\eta^*_2 = 6.7$ in our example.

4. Find the minimal and maximal values of η_1 for C1 and η_2 for C2.

   Note. These values and the respective intervals of their variation can be different for other cross sections. We find herein the following intervals for η_1 and η_2: $\eta_{1,2} \in \{6; 7.7\}$.

5. Find the safety zones S_1 and S_2. They are the areas under curves $\xi_{\rm I}$ and $\xi_{\rm II}$, and curves $\xi_{\rm I}$ and $\xi_{\rm III}$, bounded within the intervals of η_1 and η_2.

6. Introduce parameter $\Delta S = \dfrac{S_2 - S_1}{S_2}\%$, expressing the tribute of compression micro-damage.

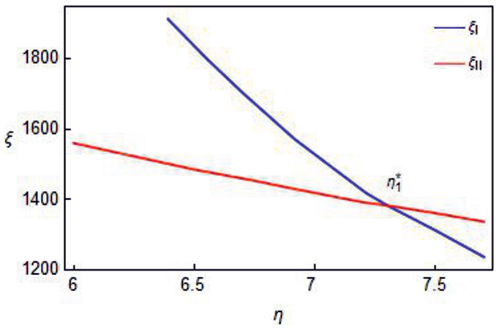


Fig. 2. Limit curves for C1

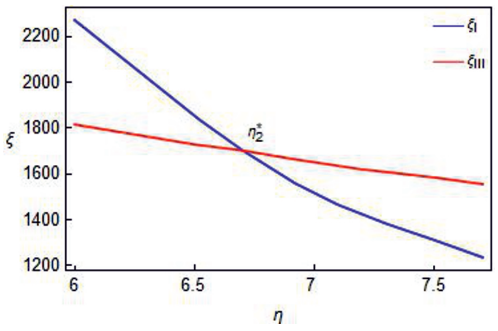


Fig. 3. Limit curves for C2

6. Results. Figures 2 and 3 show the limit curves for C1 and C2.

The areas of the sought safety zones are $S_1 = 2426$ and $S_2 = 2659$.

The value of the parameter expressing the tribute of compression micro-damage is $\Delta S \approx 9\%$.

7. Conclusion. The presented methodology for the estimation of the bearing capacity of sets of structural elements can be easily applied, and with a sufficient accuracy. The safety zones present a good estimation of the capability of the structural element to operate safely under loads exceeding the admissible norms. Zones (S_1 and S_2) characterize the degree of safety of a structure undergoing overloading.

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