

PADOVAN NUMBERS AS SUM OF TWO REPDIGITS

Merve Güney Duman, Refik Keskin*, Leman Hocaoglu*

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Abstract

Padovan sequence (P_n) is given by $P_n = P_{n-2} + P_{n-3}$ for $n \geq 3$ with initial condition $(P_0, P_1, P_2) = (1, 1, 1)$. A positive integer is called a repdigit if all of its digits are equal. In this study, we examine the terms of the Padovan sequence, which are the sum of two repdigits. It is shown that the largest term of the Padovan sequence which can be written as a sum of two repdigits is $P_{18} = 114 = 111 + 3$.

Key words: Diophantine equations, continued fraction, repdigit, linear forms in logarithms, Padovan number

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1. Introduction. A repdigit is a positive integer whose digits are all equal. Repdigits in the linear recurrence sequence have been investigated by many mathematicians. In later years, some authors investigated the terms of these sequences which are the sum of two repdigits. In [1] and [2], the authors showed that the largest of this type of Fibonacci number is $F_{20} = 6765 = 6666 + 99$ and the largest of this type of Lucas number is $L_{14} = 843 = 777 + 66$. In [3], the authors showed that the largest Pell and Pell–Lucas numbers which are a sum of two repdigits are $P_6 = 70 = 66 + 4$ and $Q_6 = 198 = 99 + 99$, respectively. In [4], the authors showed that the largest Perrin number expressible as sum of two repdigits is $R_{20} = 277 = 222 + 55$. Padovan sequence (P_n) is given by $P_n = P_{n-2} + P_{n-3}$ for $n \geq 3$, where $(P_0, P_1, P_2) = (1, 1, 1)$. P_n is called n -th Padovan number. See

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[⁵⁻⁷], etc., for more information about Padovan sequence. In this study, we show that the largest Padovan number which can be written as a sum of two repdigits is $P_{18} = 114 = 111 + 3$. In order to do this, we will solve the equation

$$(1) \quad P_k = \frac{d_1(10^n - 1)}{9} + \frac{d_2(10^m - 1)}{9},$$

where d_1, d_2, k, n , and m are positive integers with $1 \leq m \leq n, 1 \leq d_1, d_2 \leq 9$.

2. Preliminaries. Let η be an algebraic number of degree d with minimal polynomial $a_0x^d + a_1x^{d-1} + \dots + a_d = a_0 \prod_{i=1}^d (x - \eta^{(i)}) \in \mathbb{Z}[x]$, where the a_i 's are relatively prime integers with $a_0 > 0$ and the $\eta^{(i)}$'s are conjugates of η . Then the logarithmic height $h(\eta)$ of η , is defined by

$$h(\eta) = \frac{1}{d} \left(\log a_0 + \sum_{i=1}^d \log \left(\max \{ |\eta^{(i)}|, 1 \} \right) \right).$$

In particular, if $\eta = a/b$ is a rational number with $\gcd(a, b) = 1$ and $b \geq 1$, then $h(\eta) = \log(\max\{|a|, b\})$. We give some properties of the logarithmic height whose proofs can be found in [⁸]:

$$h(\eta \pm \gamma) \leq h(\eta) + h(\gamma) + \log 2,$$

$$h(\eta\gamma^{\pm 1}) \leq h(\eta) + h(\gamma),$$

$$h(\eta^m) = |m|h(\eta).$$

Let α, β, γ be the roots of the characteristic equation $x^3 - x - 1 = 0$, where

$$\alpha = \frac{\sqrt[3]{108 + 12\sqrt{69}} + \sqrt[3]{108 - 12\sqrt{69}}}{6},$$

$$\begin{aligned} \beta &= \bar{\gamma} \\ &= \frac{-\left(\sqrt[3]{108+12\sqrt{69}} + \sqrt[3]{108-12\sqrt{69}}\right) + i\sqrt{3}\left(\sqrt[3]{108+12\sqrt{69}} - \sqrt[3]{108-12\sqrt{69}}\right)}{12}. \end{aligned}$$

The Binet formula for the Padovan numbers is

$$(2) \quad P_k = a \cdot \alpha^k + b \cdot \beta^k + c \cdot \gamma^k,$$

where $23a = \alpha^3 + 7\alpha^2 + 2$, $23b = \beta^3 + 7\beta^2 + 2$, $c = \bar{b}$. Then we have $1.32 < \alpha < 1.33$, $0.86 < |\beta| = |\gamma| = \alpha^{-1/2} < 0.87$, $0.72 < a < 0.73$ and $h(a) \leq \frac{1}{3} \log 23$. In addition, the minimal polynomial of a over \mathbb{Z} is given by $23x^3 - 23x^2 + 6x - 1$, where b and c are also zeros of this equation. By induction method, it can be seen that

$\alpha^{k-2} \leq P_k \leq \alpha^{k-1}$ for $k \geq 1$ and $k \neq 3$. Let $e(k) := P_k - a \cdot \alpha^k = b \cdot \beta^k + c \cdot \gamma^k$. Then,

$$(3) \quad |e(k)| = |b \cdot \beta^k + c \cdot \gamma^k| \leq |b| \cdot \alpha^{-k/2} + |c| \cdot \alpha^{-k/2} < 2 \cdot 0.25 \cdot \alpha^{-k/2} < \frac{1}{\alpha^{k/2}},$$

for $k \geq 1$. Let $F := \mathbb{Q}(\alpha, \beta)$ be the splitting field of the polynomial ϕ over \mathbb{Q} . Then

$$|\text{Gal}(F/\mathbb{Q})| = [F : \mathbb{Q}] = 6, \quad [\mathbb{Q}(\alpha) : \mathbb{Q}] = 3$$

and

$$\text{Gal}(F/\mathbb{Q}) \simeq \{(1), (\alpha\beta), (\alpha\gamma), (\beta\gamma), (\alpha\beta\gamma), (\alpha\gamma\beta)\} \simeq S_3.$$

The following lemma is deduced from Corollary 2.3 of MATVEEV [9] (also see Theorem 9.4 in [10]).

Lemma 1. *Assume that $\gamma_1, \gamma_2, \dots, \gamma_t$ are positive real algebraic numbers in a real algebraic number field \mathbb{K} of degree D , b_1, b_2, \dots, b_t are nonzero integers, and $\Lambda := \gamma_1^{b_1} \cdots \gamma_t^{b_t} - 1$ is not zero. Then $\log |\Lambda| > -1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2 \cdot (1 + \log D) \cdot (1 + \log B) A_1 A_2 \cdots A_t$, where $B \geq \max\{|b_1|, \dots, |b_t|\}$, and $A_i \geq \max\{Dh(\gamma_i), |\log \gamma_i|, 0.16\}$ for all $i = 1, \dots, t$.*

Now we give a lemma which was proved in [11]. It is a version of the lemma given by DUJELLA and PETHÒ [12]. The lemma given in [12] is a variation of a result of BAKER and DAVENPORT [13]. It will be used to reduce the upper bound for the subscript k in equation (1). Let the function $\|\cdot\|$ denote the distance from x to the nearest integer. That is, $\|x\| = \min\{|x - n| : n \in \mathbb{Z}\}$.

Lemma 2. *Let M be a positive integer, let p/q be a convergent of the continued fraction of the irrational number γ such that $q > 6M$, and let A, B, μ be some real numbers with $A > 0$ and $B > 1$. Let $\epsilon := \|\mu q\| - M\|\gamma q\|$. If $\epsilon > 0$, then there exists no solution to the inequality $0 < |u\gamma - v + \mu| < AB^{-w}$, in positive integers u, v , and w with $u \leq M$ and $w \geq \frac{\log(Aq/\epsilon)}{\log B}$.*

Lemma 3 ([14]). *Let $a, x \in \mathbb{R}$. If $0 < a < 1$ and $|x| < a$, then $|\log(1+x)| < \frac{-\log(1-a)}{a} \cdot |x|$ and $|x| < \frac{a}{1-e^{-a}} \cdot |e^x - 1|$.*

Lemma 4 ([15]). *The largest Padovan number that can be written as a repdigit is $P_{10} = 9$.*

3. Main Theorems. In this section, we will give a lemma and a theorem about Padovan numbers and we will use Matlab for all our calculations. The following lemma will be used to prove our main theorem.

Lemma 5. *Let $10 \leq d \leq 18$. Then all positive integer solutions to equation*

$$(4) \quad P_k = \frac{d \cdot (10^n - 1)}{9}$$

are given by $P_k \in \{12, 16\}$.

Proof. Suppose that equation (4) holds. Suppose that $1 \leq k \leq 159$. In this case, it can be shown that all positive integer solutions to this equation are $P_{10} = 12$, $P_{11} = 16$. Now suppose that $k \geq 160$. Then $n \geq 19$. By (2), we write

$$P_k = a \cdot \alpha^k + b \cdot \beta^k + c \cdot \gamma^k = \frac{d \cdot (10^n - 1)}{9}.$$

We arrange the above equation as

$$(5) \quad a \cdot \alpha^k - \frac{d \cdot 10^n}{9} = -(b \cdot \beta^k + c \cdot \gamma^k) - \frac{d}{9}.$$

Taking absolute value of this equation and dividing by $a \cdot \alpha^k$, we get

$$(6) \quad \left| 1 - \frac{d \cdot 10^n \cdot \alpha^{-k}}{9a} \right| \leq \left| \frac{b \cdot \beta^k + c \cdot \gamma^k}{a \cdot \alpha^k} \right| + \left| \frac{d}{9 \cdot a \cdot \alpha^k} \right| \\ \leq \left| \frac{e(k)}{a \cdot \alpha^k} \right| + \frac{2}{a \cdot \alpha^k} \leq \frac{\alpha^{-k/2}}{a \cdot \alpha^k} + \frac{2}{a \cdot \alpha^k} < \frac{2.78}{\alpha^k},$$

by using (3). To apply Lemma 1, we take

$$(7) \quad (\Lambda, \gamma_1, \gamma_2, \gamma_3, b_1, b_2, b_3) := \left(1 - \frac{\alpha^{-k} \cdot 10^n \cdot d}{9a}, \alpha, 10, \frac{d}{9a}, -k, n, 1 \right).$$

Moreover, $\mathbb{K} = \mathbb{Q}(\gamma_1, \gamma_2, \gamma_3) = \mathbb{Q}(\alpha)$. Hence, $D = 3$. In addition, if $\Lambda = 0$, then $a \cdot \alpha^k = \frac{10^n d}{9}$. If we take the automorphism σ from both sides of this equation and apply absolute values, then we find $\left| \frac{10^n d}{9} \right| = |\sigma(a\alpha^k)| = |b\beta^k| < 1$, which is impossible. As a result, $\Lambda \neq 0$. The logarithmic heights for $\gamma_1, \gamma_2, \gamma_3$ are as follows:

$$h(\gamma_1) := h(\alpha) := \frac{\log \alpha}{3}, h(\gamma_2) := \log 10, \\ h(\gamma_3) = h\left(\frac{d}{9 \cdot a}\right) \leq h(a) + h(9) + h(d) < 5.45.$$

Then, we choose

$$(8) \quad (A_1, A_2, A_3) := (\log \alpha, \log 10^3, 16.35).$$

Moreover, we have

$$\alpha^{8(n-1)} < 10^{n-1} < \frac{1}{9}(10^n - 1) < \frac{d(10^n - 1)}{9} = P_k \leq \alpha^{k-1},$$

which implies that $n < 8n < k + 7$. Also, since $B \geq \max\{|k|, |n|, 1\}$, we can take $B := k + 7$. By using (6), (7), (8), and Lemma 1, we obtain

$$k \log \alpha - \log(2.78) < 8.59 \cdot 10^{13} \cdot (1 + \log(k + 7)).$$

Then, it can be shown that $160 \leq k < 1.161 \cdot 10^{16}$ and $n < 1.452 \cdot 10^{15}$. In order to use Lemma 2, let

$$(9) \quad z := \log 10 \cdot n - k \cdot \log \alpha + \log \left(\frac{d}{9a} \right).$$

Then, by using (6), we can write $|x| = |e^z - 1| < \frac{2.78}{\alpha^k} < 0.1$ for $k \geq 160$. Now take $a := 0.1$ to use Lemma 3. So, we obtain the inequalities

$$(10) \quad |z| = |\log(x + 1)| < \frac{\log(10/9)}{(1/10)} \cdot \frac{2.78}{\alpha^k} < 2.93 \cdot \alpha^{-k}.$$

From (9) and (10), we get

$$(11) \quad 0 < \left| n \left(\frac{\log 10}{\log \alpha} \right) - k + \frac{\log(d/(9a))}{\log \alpha} \right| < 10.42 \cdot \alpha^{-k}.$$

If we take $\gamma := \frac{\log 10}{\log \alpha} \notin \mathbb{Q}$, $M := 1.452 \cdot 10^{15}$ and $\mu := \frac{\log(d/9a)}{\log \alpha}$, then we see that q_{39} is the denominator of the 39th convergent of γ exceeding $6M$. For $10 \leq d \leq 18$, we have $0.08 < \epsilon = \epsilon(\mu) = \|\mu q_{39}\| - M \|\gamma q_{39}\| < 0.499$. In Lemma 2, suppose $(A, B, w) := (10.42, \alpha, k)$. In this case, there is no solution to inequality (11) if

$$\log(A \cdot q_{39}/\epsilon) / \log B < 149.1 < k.$$

This implies that $k \leq 149$. But, this is impossible since $k \geq 160$. □

Theorem 6. *If P_k is expressible as sum of two repdigits, then*

$$P_k \in \{2, 3, 4, 5, 7, 9, 12, 16, 28, 37, 49, 86, 114\}.$$

Proof. Suppose that equation (1) holds. Suppose $1 \leq k < 210$. In this case, solutions of this equation are given by

$$\begin{aligned} P_3 = P_4 = 2 &= 1 + 1, & P_5 = 3 &= 2 + 1, & P_6 = 4 &= 1 + 3 = 2 + 2, \\ P_8 = 7 = 1 + 6 &= 2 + 5 = 3 + 4, & P_9 = 9 &= 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5, \\ P_{10} = 12 = 11 + 1 &= 3 + 9 = 4 + 8 = 5 + 7 = 6 + 6, \\ P_{11} = 16 = 11 + 5 &= 7 + 9 = 8 + 8, & P_{13} = 28 &= 22 + 6, & P_{14} = 37 &= 33 + 4, \\ P_{15} = 49 = 44 + 5, & P_7 = 5 &= 1 + 4 = 2 + 3, & P_{17} = 86 &= 77 + 9, \\ P_{18} = 114 &= 111 + 3. \end{aligned}$$

Now suppose that $k \geq 210$. By (1) and (2), we write

$$P_k = a \cdot \alpha^k + b \cdot \beta^k + c \cdot \gamma^k = \frac{d_1 \cdot (10^n - 1)}{9} + \frac{d_2 \cdot (10^m - 1)}{9}.$$

We arrange the above equation in two different ways as

$$(12) \quad 9 \cdot a \cdot \alpha^k - d_1 \cdot 10^n = -9(b \cdot \beta^k + c \cdot \gamma^k) + d_2 \cdot 10^m - (d_1 + d_2),$$

and

$$(13) \quad a \cdot \alpha^k - \frac{d_1 \cdot 10^n + d_2 \cdot 10^m}{9} = -(b \cdot \beta^k + c \cdot \gamma^k) - \frac{(d_1 + d_2)}{9}.$$

Taking absolute value of these equations and arranging them, for $k \geq 210$, we get

$$(14) \quad \left| \frac{9 \cdot a \cdot \alpha^k}{d_1 \cdot 10^n} - 1 \right| \leq \frac{9|b \cdot \beta^k + c \cdot \gamma^k|}{d_1 10^n} + \left| \frac{d_2 10^m - d_1 - d_2}{d_1 10^n} \right| \\ \leq \frac{9\alpha^{-k/2}}{10^{n-m+1}} + \frac{9}{10^{n-m}} < \frac{9.11}{10^{n-m}},$$

and

$$(15) \quad \left| 1 - \frac{(d_1 + d_2 10^{m-n}) \cdot 10^n \cdot \alpha^{-k}}{9a} \right| \leq \left| \frac{(b \cdot \beta^k + c \cdot \gamma^k)}{a \cdot \alpha^k} \right| + \left| \frac{(d_1 + d_2)}{9 \cdot a \cdot \alpha^k} \right| \\ \leq \frac{\alpha^{-k/2}}{a \cdot \alpha^k} + \frac{18}{9 \cdot a \cdot \alpha^k} < \frac{2.77}{\alpha^k},$$

by (3). To apply Lemma 1, we take

$$(16) \quad (\Lambda_1, \gamma_1, \gamma_2, \gamma_3, b_1, b_2, b_3) := \left(\frac{9a\alpha^k 10^{-n}}{d_1} - 1, \alpha, 10, \frac{9a}{d_1}, k, -n, 1 \right),$$

$$(17) \quad (\gamma'_1, \gamma'_2, \gamma'_3) := \left(\alpha, 10, \frac{(d_1 + d_2 10^{m-n})}{9a} \right),$$

$$(18) \quad (\Lambda_2, b'_1, b'_2, b'_3) := \left(1 - \frac{\alpha^{-k} 10^n (d_1 + d_2 10^{m-n})}{9a}, -k, n, 1 \right).$$

Moreover, we have $\mathbb{K} = \mathbb{Q}(\gamma_1, \gamma_2, \gamma_3) = \mathbb{Q}(\gamma'_1, \gamma'_2, \gamma'_3) = \mathbb{Q}(\alpha)$. Thus, $D = 3$. Additionally, if $\Lambda_1 = 0$, then $a \cdot \alpha^k = \frac{10^n d_1}{9}$ and if $\Lambda_2 = 0$, then $a \cdot \alpha^k = \frac{10^n (d_1 + d_2 10^{m-n})}{9}$. Taking the automorphism σ from both sides of these equations and considering absolute values, we find that $\left| \frac{10^n d_1}{9} \right| = |\sigma(a\alpha^k)| = |b\beta^k| < 1$, and $\left| \frac{10^n (d_1 + d_2 10^{m-n})}{9} \right| = |\sigma(a\alpha^k)| = |b\beta^k| < 1$, respectively. These are

impossible. As a result, $\Lambda_1 \neq 0$ and $\Lambda_2 \neq 0$. The logarithmic heights for $\gamma_1, \gamma_2, \gamma_3, \gamma'_1, \gamma'_2, \gamma'_3$ are as follows:

$$\begin{aligned} h(\gamma_1) &= h(\gamma'_1) := h(\alpha) := \frac{\log \alpha}{3}, h(\gamma_2) = h(\gamma'_2) := \log 10, \\ h(\gamma_3) &= h\left(\frac{a \cdot 9}{d_1}\right) \leq h(a) + h(9) + h(d_1) \leq 2 \cdot \log 9 + \frac{1}{3} \log 23 < 5.44, \\ h(\gamma'_3) &= h\left(\frac{d_1 + d_2 10^{m-n}}{9 \cdot a}\right) \leq \log 10 \cdot (n - m) + 8.34. \end{aligned}$$

Then, we can take

$$(19) \quad (A_1, A_2, A_3, A'_1, A'_2, A'_3) \\ := (\log \alpha, \log 10^3, 16.32, \log \alpha, \log 10^3, 25.02 + 3(n - m) \log 10).$$

Moreover, we have the inequality

$$\alpha^{8(n-1)} < 10^{n-1} < \frac{d_1(10^n - 1)}{9} + \frac{d_2(10^m - 1)}{9} = P_k \leq \alpha^{k-1},$$

which implies that $n < 8n < k + 7$. Also, since $B \geq \max\{|k|, |n|, 1\}$, we can take $B := k + 7$. By using Lemma 1, (14), (16), (19), we write

$$(20) \quad (n - m) \log 10 < 8.59 \cdot 10^{13} \cdot (1 + \log(k + 7)) + \log(9.11).$$

Similarly, by using (15), (17), (18), (19) and Lemma 1, we obtain

$$(21) \quad k \log \alpha - \log(2.77) < 5.27 \cdot 10^{12} \cdot (1 + \log(k + 7)) (25.02 + 3(n - m) \log 10).$$

From (20) and (21), it can be shown that $217 \leq k + 7 < 2.61 \cdot 10^{31}$ and $n < k + 7 < 2.61 \cdot 10^{31}$. Now, in order to use Lemma 2, let

$$(22) \quad z_1 := -n \cdot \log 10 + k \cdot \log \alpha + \log\left(9 \frac{a}{d_1}\right)$$

and

$$(23) \quad z_2 := \log 10 \cdot n - k \cdot \log \alpha + \log(d_1 + d_2 10^{m-n})/9a.$$

Suppose that $n = m$. If $0 < d_1 + d_2 \leq 9$, then P_k is a repdigit and if $10 \leq d_1 + d_2 = d \leq 18$, then we get $P_k = \frac{d \cdot (10^n - 1)}{9}$. These are impossible by Lemma 4 and Lemma 5. Therefore, assume that $n - m \geq 1$. Then, for $k \geq 210$, we get

$$|x| = |e^{z_1} - 1| < \frac{9.11}{10^{n-m}} < 0.99 \quad \text{and} \quad |x'| = |e^{z_2} - 1| < \frac{2.77}{\alpha^k} < 0.1$$

by (14) and (15). Now take $a := 0.99$ and $a' := 0.1$ to use Lemma 3. By (22) and (23), we write

$$(24) \quad 0 < \left| k \left(\frac{\log \alpha}{\log 10} \right) - n + \left(\frac{\log(9a/d_1)}{\log 10} \right) \right| < 18.5 \cdot 10^{m-n},$$

and

$$(25) \quad 0 < \left| n \left(\frac{\log 10}{\log \alpha} \right) - k + \frac{\log((d_1 + d_2 10^{m-n})/(9a))}{\log \alpha} \right| < 10.4 \cdot \alpha^{-k}.$$

If we take $\gamma := \frac{\log \alpha}{\log 10} \notin \mathbb{Q}$, $\gamma' := \frac{\log 10}{\log \alpha} \notin \mathbb{Q}$ and $M := 2.61 \cdot 10^{31}$, then q_{71} is the denominator of the 71st convergent of γ exceeding $6M$. We apply Lemma 2 for $\gamma := \frac{\log \alpha}{\log 10}$ and take $\mu := \log(9a/d_1)/\log 10$. It can be shown that $0.008 < \epsilon = \epsilon(\mu) := \|\mu q_{71}\| - M \|\gamma q_{71}\| < 0.45$ for $1 \leq d_1 \leq 9$. In Lemma 2, take $(A, B, w) := (18.5, 10, n - m)$. In this case, there is no solution to inequality (24) if

$$\log(18.5 \cdot q_{71}/\epsilon)/\log 10 < 36.4 < n - m.$$

Therefore, we get that $n - m \leq 36$. Also, from (21), it follows that $k < 2.1 \cdot 10^{17}$ and therefore $n < 2.1 \cdot 10^{17}$. Similarly, we can take $\gamma' := \frac{\log 10}{\log \alpha} \notin \mathbb{Q}$, $M' := 2.1 \cdot 10^{17}$,

and $\mu := \frac{\log(d_1 + d_2 10^{m-n})/9a}{\log \alpha}$. It can be seen that q_{43} is the denominator of the 43rd convergent of γ' exceeding $6M'$. For $1 \leq n - m \leq 36$ and $1 \leq d_1, d_2 \leq 9$, we have $0.0001 < \epsilon = \epsilon(\mu) = \|\mu q_{43}\| - M' \|\gamma' q_{43}\| < 0.4999$. In Lemma 2, suppose that $(A, B, w) := (10.4, \alpha, k)$. In this case, there is no solution to inequality (25) if

$$\log(A \cdot q_{43}/\epsilon)/\log B < 202.6 < k.$$

This implies that $k \leq 202$, which is impossible since $k \geq 210$. This completes the proof. \square

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Sakarya University of Applied Sciences
Sakarya, Turkey
e-mail: merveduman@subu.edu.tr

**Sakarya University*
Mathematics Department
Sakarya, Turkey
e-mail: rkeskin@sakarya.edu.tr
lemanhocaoglu06@hotmail.com