

MODELLING AND FEEDBACK CONTROL
FOR REVERSING A NONHOLONOMIC
MOBILE ROBOT PLATOON

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Abstract

This paper addresses the control problem of a platoon of nonholonomic mobile robots in the case of backward motion of the leading robot. By using a virtual non-static with respect to the lead robot reference frame as a target, a kinematic model of the robot platoon in error coordinates is developed. A nonlinear feedback controller based on the entire platoon model is designed by means of high-gain control. Asymptotic stability property of the closed-loop system is established using Lyapunov theory. For backward motion of the leading robot along a circular path, at steady-state, the two-robot platoon should travel concentric arcs of same radii with prescribed inter-robot spacing. The performance of the proposed tracking controller is illustrated through numerical simulations.

Key words: mobile robot, reverse platooning, mathematical model, high-gain control

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1. Introduction. In recent years, cooperative robotics has attracted considerable interest [1]. The concept of mobile robot platooning consists in grouping robots, where each robot has to follow the preceding robot in a convoy-like fashion, and in the same time to maintain a desired distance between them. Platooning applications in different fields include military convoys [2], platoon formations of commercial or public urban vehicles [3], fleets of AGVs for warehouse automation [4–6], etc. Platooning has been a research topic on autonomous vehicles and mobile robots from different perspectives. Concerning the platoon motion control, two main approaches have been used: path following and trajectory tracking [7]. In the path following approach, the longitudinal and the lateral control are usually treated separately. A disadvantage of this approach is that the lateral control is, in most cases, infrastructure-dependent (for example lane markings [8,9]). In the trajectory tracking or direct vehicle following approach, the current position and orientation of the preceding vehicle, as well as the desired inter-vehicle spacing are used for motion control design. Several design tools for platooning control have been reported in the literature, among them predominate Lyapunov-based design [10] and feedback linearization [7,11]. A specific feature of the controllers, which are designed via feedback linearization techniques, is that in most cases, they are look-ahead type tracking controllers, for which the design procedure is based on a reduced-order system model. The internal (unobservable) dynamics, which has not been considered in the feedback control design, is stable only if the lead robot velocity is positive and does not change its sign, that is, while the lead robot is moving forward followed by the other robots. But, in case of backward movement of the leading robot, then its velocity changes the sign, and the internal dynamics becomes unstable, resulting in the lead and following robot being folded up against one another.

Until now, there have been very few papers dealing with the platoon backward movement of autonomous vehicles or mobile robots [12], however, the importance of integrating the ability of backward driving arises from many practical situations in structured or natural terrains, for example, in cases of avoiding collision with other vehicles, objects or pedestrians [13]. One of the challenges in designing controllers for autonomous robot following, where the leading robot is driven in reverse, is to decide the desired trajectory of the following robot. When the lead robot is moving forward, the following robot is behind the leader and needs in some degree to “retrace” its path at a desired prescribed distance. However, if the leading robot is moving backward, the following robot has to move in front of the leader, and the control problem formulation is not straightforward, as different tracking scenarios are possible.

In this paper, we consider the problem of modelling and backward driving control of a two unicycle-type mobile robot platoon. We assume that the relative position and orientation of the following robot with respect to the lead robot is obtained from onboard sensors (lidar-based). It is also assumed that lead robot

linear and angular velocities are known via inter-robot communication. The desired spacing is a function of the leader linear and angular velocity, and an a priori known desired inter-robot spacing, which is defined for the case of a straight line maneuver. A virtual reference frame located behind the leading robot is assigned using leader linear and angular velocities and the desired inter-robot distance. This non-static with respect to the leader reference frame contains the desired platoon pattern and is used for formulation of the tracking problem during backward driving. The underlying idea is the following robot to track the virtual reference frame as a target, and as a result, during constant velocity maneuvers of the leading robot, at steady state, the two-robot platoon should travel concentric arcs of same radii with prescribed inter-robot spacing. The main contribution of this paper is the design of a backward motion controller for the following mobile robot, which is based on the entire platoon model. The rest of the paper is organized, as follows. In Section 2, the platoon model based on the relative robot kinematics is presented and the control objective is formulated. In Section 3, the tracking controller for backward driving of a two-robot platoon is presented. Section 4 contains simulation results which illustrate the effectiveness of the proposed controller. Conclusions are presented in Section 5.

2. Problem description. 2.1. Platoon model. In this paper, we consider a platooning scenario, where the lead robot is moving backward and the following robot is situated in front of the leader. Both robots are assumed to be nonholonomic unicycle-type wheeled mobile robots. A schematic illustration of the platoon geometry in the plane is shown in Fig. 1.

In order to describe the platoon kinematic model, the following coordinate systems are defined: An inertial coordinate system Fxy in the plane to travel; Robot coordinate systems $P_i x_i y_i$ for each robot i , ($i = 1, 2$), located at the mid-point of the wheel axle with x_i axis directed along the longitudinal base of the robot, where the index $i = 1$ is used to denote the lead robot, and $i = 2$ – for the follower robot. The following notations are used in this paper: ${}^F \mathbf{p}_i = [x_i, y_i]^T \in \mathbb{R}^2$, ($i = 1, 2$), is a vector that represents the coordinates of a point P_i with respect to the inertial frame Fxy ; ${}^{P_1} \mathbf{p}_2 = [e_{xm}, e_{ym}]^T \in \mathbb{R}^2$ is a vector that denotes the coordinates of point P_2 in the coordinate system $P_1 x_1 y_1$; ${}^F \dot{\mathbf{p}}_{P_i} = [\dot{x}_i, \dot{y}_i]^T \in \mathbb{R}^2$ denotes a vector of the components of the velocity of point P_i with respect to the inertial frame Fxy . ${}^{P_i} \dot{\mathbf{p}}_{P_i} = [v_{ix}, v_{iy}]^T \in \mathbb{R}^2$ is a vector of the components of the velocity of point P_i with respect to the inertial frame Fxy and expressed in a moving body coordinate frame $P_i x_i y_i$. The orientation of each robot in the inertial coordinate system Fxy is denoted by $\theta_i \in S^1$, ($i = 1, 2$). The desired inter-robot distance is not constant and is assigned to be equal to $2h = |RP_1|$, in the case of straight line motion of the robot platoon (Fig. 1).

In order to assign the desired position of the following robot during backward motion of the robot platoon, a moving virtual reference coordinate system $Rx_R y_R$ (Fig. 1) is defined as a tracking objective for the following robot, such that the

location of its centre R in the coordinate system $P_1x_1y_1$ of the lead robot is specified as

$$(1) \quad {}^{P_1}\mathbf{p}_R = [{}^{P_1}x_R, {}^{P_1}y_R]^T = [-|h|(1 + \cos \varphi), -|h| \sin \varphi]^T \in \mathbb{R}^2.$$

In (1), the angle $\varphi = \varphi(v_{1x}, \omega_1)$ is determined from the linear (v_1) and angular (ω_1) velocity of the lead robot. The virtual reference frame Rx_Ry_R is assigned in a way, that its axes remain parallel to the axes of the lead robot reference frame $P_1x_1y_1$. It should be noted that point R is located on the arc of circle with known radius (radius of curvature) $\rho = v_{1x}/\omega_1$, that passes through the guide point P_1 of the lead robot. The location of the reference point R can be interpreted as a desired position of the reference point P_2 of the following vehicle.

The coordinates ${}^R\mathbf{p}_2$ of point P_2 in the reference coordinate system Rx_Ry_R are denoted by ${}^R\mathbf{p}_2 = [e_x, e_y]^T \in \mathbb{R}^{2T}$. The relative kinematics between the points P_2 and R is defined with respect to the moving reference frame Rx_Ry_R . In order to obtain the coordinates of point P_2 in the reference coordinate system Rx_Ry_R (${}^R\mathbf{p}_2$), the following relationships between coordinates of P_2 , P_1 and R are used

$$(2) \quad {}^{P_1}\mathbf{p}_2 = {}^{P_1}\mathbf{p}_R + {}^R\mathbf{p}_2,$$

$$(3) \quad {}^F\mathbf{p}_2 = {}^F\mathbf{p}_1 + R(\theta_1){}^{P_1}\mathbf{p}_2,$$

where $R(\theta_1) = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{bmatrix} \in SO(2)$ represents an orthogonal rotation

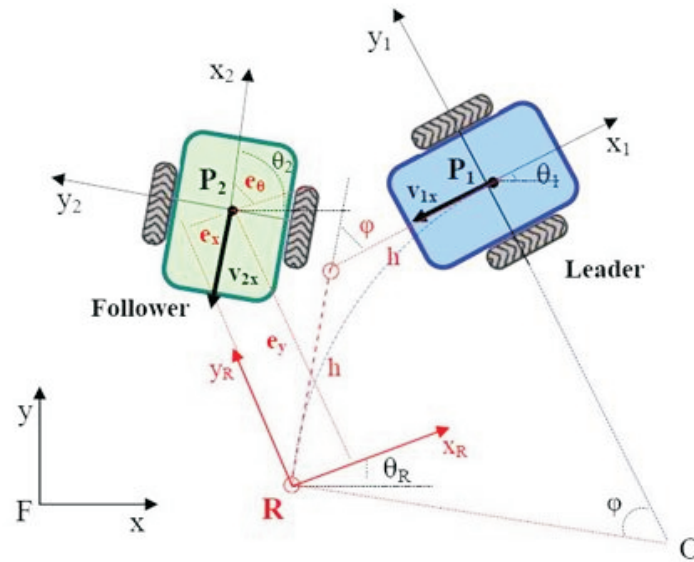


Fig. 1. Platoon configuration and description of the platoon geometry in the plane

matrix of angle θ_1 .

Substituting (2) in (3), we get

$$(4) \quad {}^R\mathbf{p}_2 = R^T(\theta_1) ({}^F\mathbf{p}_2 - {}^F\mathbf{p}_1) - {}^{P_1}\mathbf{p}_R.$$

The orientation of the following robot with respect to the reference coordinate system is

$$(5) \quad \begin{aligned} e_\theta &= \theta_2 - \theta_R \\ &= \theta_2 - \theta_1 \end{aligned} ,$$

where θ_R is the orientation of the reference frame Rx_Ry_R with respect to the inertial frame Fxy .

Remark. It should be pointed out that the orientation e_θ of the following robot ($P_2x_2y_2$) with respect to the reference coordinate system (Rx_Ry_R) is the same as its orientation e_{θ_m} with respect to the body-linked coordinate system of the lead robot ($P_1x_1y_1$), i.e., $e_\theta = e_{\theta_m}$, since Rx_Ry_R is assigned in a way that its coordinate axes remain parallel all the time to the axes of the coordinate system $P_1x_1y_1$, and because of that $\theta_R = \theta_1$. $\square$

Differentiating (4) and using the expressions relating the velocity components of points P_1 and P_2 in a global and local coordinate system

$$(6) \quad {}^F\dot{\mathbf{p}}_{P_2} = R(\theta_2)^{P_2}\dot{\mathbf{p}}_{P_2}, \quad {}^F\dot{\mathbf{p}}_{P_1} = R(\theta_1)^{P_1}\dot{\mathbf{p}}_{P_1},$$

after some work, we obtain

$$(7) \quad {}^R\dot{\mathbf{p}}_2 = S^T(\omega_1) ({}^R\mathbf{p}_2 + {}^{P_1}\mathbf{p}_R) - {}^{P_1}\mathbf{v}_{P_1} + R(e_\theta)^{P_2}\mathbf{v}_{P_2} - {}^{P_1}\mathbf{v}_R,$$

where ${}^{P_i}\mathbf{v}_{P_i} := {}^{P_i}\dot{\mathbf{p}}_{P_i}$ ($v_{iy} = 0$ according to the nonholonomic constraints, $i = 1, 2$); $\omega_i := \dot{\theta}_i \in \mathbb{R}^1$ denotes the angular velocity of each robot, ($i = 1, 2$); $R(e_\theta) \in SO(2)$; $S(\omega_1) \in SS(2)$ is a skew symmetric matrix; ${}^{P_1}\mathbf{v}_R := {}^{P_1}\dot{\mathbf{p}}_R = \begin{bmatrix} -h\omega_\varphi \sin \varphi \\ h\omega_\varphi \cos \varphi \end{bmatrix} \in \mathbb{R}^2$; and $\omega_\varphi = d\varphi/dt$.

In addition, differentiating (5), and substituting $\omega_R := \dot{\theta}_R$ with ω_2 , we obtain

$$(8) \quad \dot{e}_\theta = \omega_2 - \omega_1.$$

Equations (7) and (8) form the error kinematic model of the two-robot platoon.

2.2. Control objective. We consider a robot platooning where a mobile robot follows the lead robot during backward motion of the leader at a prescribed distance in front of him. The platoon trajectory is determined by the leading robot.

The leader linear and angular velocities (v_{1x} , ω_1) are assumed to be known for the following robot via inter-robot communication. It is also assumed that the

following robot is equipped with a sensor (2D laser range-finder), and in that way, the relative position ${}^{P_1}\mathbf{p}_2$ and orientation ($e_{\theta m}$) with respect to the leading robot are measured and known.

The desired position of the following robot is associated with a virtual reference frame Rx_Ry_R behind the leading robot (Fig. 1), and contains the desired platooning pattern – its position with respect to the leading robot is not static and is specified via the leader velocities and the assigned inter-robot distance, as given in (1). The error coordinates $\mathbf{e} := {}^R\mathbf{p}_2 = [e_x, e_y]^T$ (the coordinates of point P_2 in the virtual reference system Rx_Ry_R) are obtained from (2) via (1). The orientation e_θ of the following robot with respect to the moving virtual reference frame Rx_Ry_R is the same as its orientation $e_{\theta m}$ with respect to the lead robot.

In this paper, we assume slow motion constant linear and angular velocity maneuvers of the lead robot, and we are interested in a tracking scenario where at steady-state, the two-robot platoon should travel concentric arcs of same radii with prescribed inter-robot distance depending of the linear and angular velocities of the leading robot. The control problem is formulated as a trajectory-tracking problem, and the control objective is to track the moving virtual reference point R with the reference point P_2 of the following robot, i.e., to asymptotically stabilize the error coordinates ${}^R\mathbf{p}_2$ to zero. Due to the specific assignment of the reference coordinate system Rx_Ry_R with axes parallel to the axes of the leading robot coordinate system $P_1x_1y_1$, the relative orientation e_θ between the following robot ($P_2x_2y_2$) and reference frame Rx_Ry_R will converge to a constant value $\bar{e}_\theta = \bar{\varphi} = -2a \tan\left(h \frac{\bar{\omega}_1}{\bar{v}_1}\right)$, which is in general, not equal to zero, except in the case of straight line motion of the leading robot.

3. Platoon control design. We will consider the problem of motion control design for the following robot during constant velocity maneuvers in the reverse direction of the leading robot. To this end, combining (7) and (8), and setting ${}^{P_1}\mathbf{v}_R = 0$, the error kinematic model can be written as follows

$$\begin{aligned} \dot{e}_x &= -v_{1x} + v_{2x} \cos e_\theta + \omega_1 (e_y - h \sin \varphi) \\ \dot{e}_y &= -v_{2x} \sin e_\theta - \omega_1 [e_x - h (1 + \cos \varphi)] \quad . \\ \dot{e}_\theta &= -\omega_1 + \omega_2 \end{aligned} \quad (9)$$

We consider the following change of variable

$$\mu = \tan e_\theta, \quad (10)$$

and control inputs

$$\begin{aligned} u_1 &= v_{2x} \cos e_\theta \\ u_2 &= \frac{-\omega_1 + \omega_2}{\cos^2 e_\theta} \quad . \end{aligned} \quad (11)$$

The system (9) in the new variables and control inputs is obtained in the form

$$(12) \quad \begin{aligned} \dot{e}_x &= -v_{1x} + \omega_1 (e_y - h \sin \varphi) + u_1 \\ \dot{e}_y &= -\omega_1 [e_x - h (1 + \cos \varphi)] + u_1 \mu \\ \dot{\mu} &= u_2 \end{aligned}$$

For the first equation of (12), the following control law is proposed

$$(13) \quad u_1 = v_{1x} - \omega_1 (e_y - h \sin \varphi) - k_x e_x,$$

where k_x [1/s] is a positive gain.

Using the state variable μ as a control input, the following feedback control for the second equation of (12) is proposed

$$(14) \quad \mu^d := \mu = \frac{1}{u_1} \{ \omega_1 [e_x - h (1 + \cos \varphi)] - k_y e_y \},$$

where k_y [1/s] is a positive gain. For the third equation of (12), we use high-gain control in the form

$$(15) \quad u_2 = -K (\mu - \mu^d),$$

where K [1/s] is a positive gain.

Using the control (13), (14) and (15), the closed-loop system is obtained as follows

$$(16) \quad \begin{aligned} \dot{e} &= f(e) + g(e)\mu \\ \dot{\mu} &= -K(\mu - \mu^d) \end{aligned},$$

where

$$\dot{e} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix}, \quad f(e) = \begin{bmatrix} -k_x e_x \\ -\omega_1 [e_x - h (1 + \cos \varphi)] \end{bmatrix}, \quad g(e) = \begin{bmatrix} 0 \\ u_1 \end{bmatrix}.$$

Applying the change of variable

$$(17) \quad \eta = \mu - \mu^d,$$

the closed-loop system (16) can be written in the form

$$(18) \quad \begin{aligned} \dot{e} &= F(e) + g(e)\eta \\ \dot{\eta} &= -K\eta + \eta^d \end{aligned},$$

where

$$F(e) = f(e) + g(e)\mu^d, \quad \eta^d = -\frac{d\mu^d}{dt} = -\frac{\partial \mu^d}{\partial e} [F(e) + g(e)\eta].$$

The stability analysis of (18) is based on Proposition 6.3 [14]. A positive-definite quadratic Lyapunov function candidate is chosen in the form

$$(19) \quad V_H(e, \eta) = V_e(e) + \frac{1}{2}\eta^2,$$

where $V_e(e) = \frac{1}{2}(e_x^2 + e_y^2)$.

The derivative of (19) along the trajectories of (18) is given by

$$(20) \quad \begin{aligned} \dot{V}_H &= \frac{\partial V_e}{\partial e} F(e) + \eta \left[\frac{\partial V_e}{\partial e} g(e) + \eta^d \right] - K\eta^2 \\ &\leq -a_1 \|e\|^2 + \eta a_2 \|e\| - K\eta^2 \end{aligned}$$

It can be shown that for $K > \frac{a_2^2}{4a_1}$, where a_1 and a_2 are positive constants, $\dot{V}_H < 0$, and the high-gain control (15) achieves semi-global stabilization of $(e, \eta) = 0$, which means that the region of attraction of $(e, \eta) = (0, 0)$ can be arbitrarily increased by increasing the value of constant K [14].

4. Simulation results. A number of simulations were performed in order to verify the developed platoon model, and evaluate the effectiveness of the proposed high-gain controller for the following robot, during backward motion of the lead robot. For the simulations, the dimensions of a unicycle mobile robot with one meter in length and width were selected. The desired inter-robot spacing between the reference points P_1 and P_2 of leading and following robots was set to be equal to $2h = 2$ m in a straight-line motion, which corresponds to 1 m spacing between the robot bumpers. The controller gains were chosen to be $k_x = k_y = 0.6$ and $K = 5$. The lead robot trajectory for the simulations consists of a sequence of a circular arc with linear velocity $v_{1x} = -0.1$ m/s and angular velocity $\omega_1 = 0.1$ rad/s, followed by a circular arc with linear velocity $v_{1x} = -1$ m/s and angular velocity $\omega_1 = -0.1$ rad/s, and in the end, a straight line with linear velocity $v_{1x} = -1$ m/s. The initial error coordinates were chosen to be $e_x(0) = 0.5$ m, $e_y(0) = 0$ m, and $\eta(0) = -0.2$.

Evolution in time of the error coordinates e_x (red line) e_y (magenta) and η (blue line) are depicted in Fig. 2(a). It can be seen that the errors coordinates $(e_x(t), e_y(t), \eta(t))$ converge asymptotically to zero, which is in conformity with the stability analysis in Section 3. The path of the leading robot (blue line) and the following robot (green line) in the plane drawn by the robot reference points during backward motion of the platoon, are depicted in Fig. 2(b). It can be seen that, due to the specific feature of the assignment of the moving reference frame Rx_Ry_R , during the backward motion along a curvilinear path of the lead robot, at steady-state, the two-robot platoon is travelling concentric arcs of same radii. Evolution in time of the inter-robot distance expressed as the distance between the reference points P_1 and P_2 of the leading and following robot, respectively,

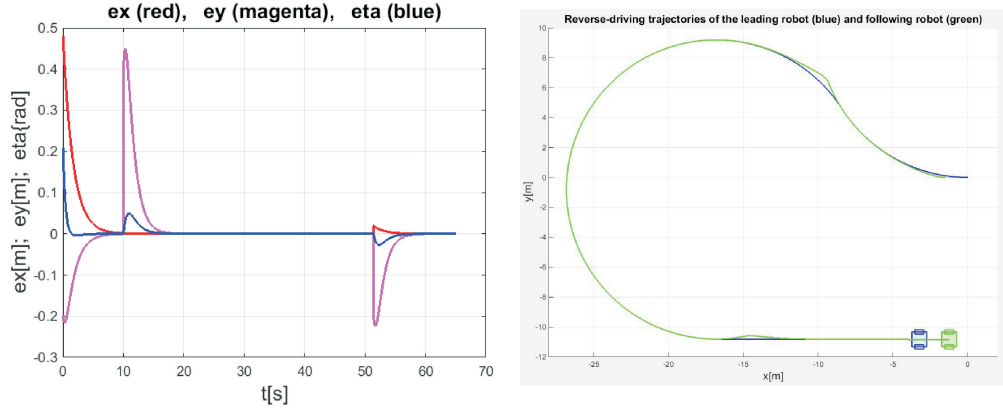


Fig. 2. (a) Evolution in time of the error coordinates: e_x (red line), e_y (magenta) and η (blue line); (b) Planar path during backward motion of the leading robot (blue line) and following robot (green line)

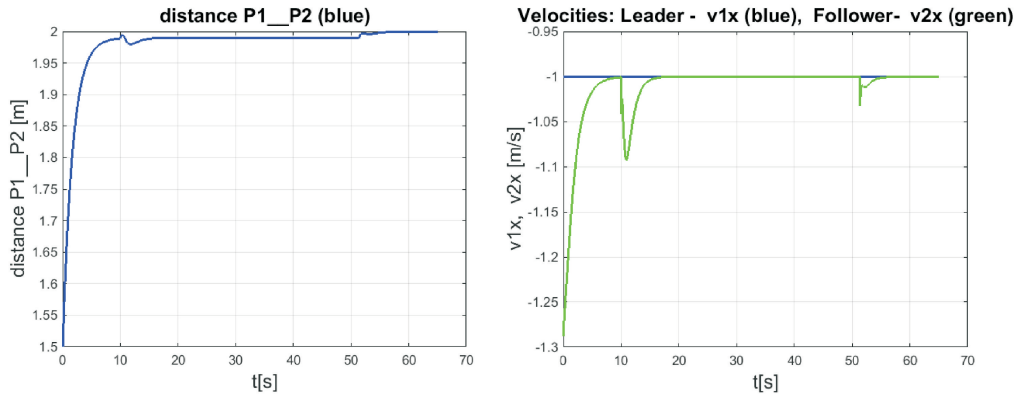


Fig. 3. (a) Evolution in time of the inter-robot distance, expressed as the distance between the reference points P_1 (leading robot) and P_2 (following robot); (b) Evolution in time of the lead robot linear velocity v_{1x} (blue line) and the linear velocity of the following robot v_{2x} (green line)

is shown in Fig. 3(a). In Fig. 3(b), are depicted the evolution in time of the velocity of the lead robot (blue line) and the following robot (green line). The simulation results confirmed the validity of the designed high-gain controller, and are in accordance with the stability analysis presented in Section 3.

5. Conclusion. In this paper, the control of a platoon of nonholonomic mobile robots in the case of backward motion of the leading robot has been considered. Assigning a virtual non-static with respect to the lead robot reference frame, a kinematic model of the two-robot platoon in error coordinates was developed. Using high-gain control design framework, a nonlinear controller was synthesized based on the entire platoon model. Asymptotic stability property of the closed-loop system in error coordinates was established using Lyapunov the-

ory. For backward motion of the leading robot, at steady state, the two-robot platoon should travel concentric arcs of same radii with prescribed inter-robot spacing. The performance of the proposed tracking controller has been illustrated through numerical simulations.

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