

GERMAN EQUATORIAL MOUNT EMPLOYING
OPTIMIZED OFFSET
SLIDER CRANK MECHANISM

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Abstract

This article comments on the design of an actuator mechanism employed in a German equatorial mount for the purpose of establishing the right ascension rotation. The German equatorial mount is used for rotating an optical solar telescope. Both, the German equatorial mount and the telescope are being built at the Space Research and Technology Institute – Bulgarian Academy of Sciences.

The device is a variant of the offset slider crank mechanism having certain properties put as prerequisites aiming to satisfy specific needs of the equatorial mount. One such requirement is the angle of rotation being not less than 180 degrees. Another condition sought is to minimize the parameter of the device, namely the angle between the connecting rod and crank pin tangent. The latter minimization leads to lessening of forces exerted on the mechanism's parts.

After a short introduction of the subject and motivation, the article establishes the problem, then offers a solution. Finally, conclusions are drawn.

Key words: German equatorial mount, solar optical telescope

Introduction. There are two major categories of telescope mounts: altitude-azimuth mounts and equatorial mounts. Altitude-azimuth mounts (AAM) have two degrees of freedom allowing the telescope to rotate on both a vertical and a horizontal axis, i.e. up-down and left-right. In respect to the horizon, this rotation is parallel and perpendicular to it, rendering the AAM simple and intuitive to use.

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The AAMs are very convenient for observational telescopes that are rotated by hand. Another benefit is that the eyepiece is always in a comfortable position. On the other hand, if the observer needs to track an object, either for observational reasons or for photography, the process of using this type of mount becomes more difficult. AAMs equipped with automatic tracking do exist, but they expose another drawback – the sky rotates in the eyepiece/camera.

The equatorial mount (EM) also has two degrees of freedom – this type of mount supports two rotations along two perpendicular axes. The more important rotation is the right ascension rotation which turns the telescope along an axis parallel to Earth's axis of rotation. In other words, the right ascension rotation of the equatorial mount counteracts Earth's rotation. As a result, the sky, in the eyepiece or camera, seems not to move nor rotate. The other axis of rotation of the equatorial mount is the declination axis, allowing the mount to position the telescope to any direction in the sky. Equatorial mounts are implemented mostly for automatic tracking. Disadvantages of the EM include non-intuitive pointing and more weight. When employed for observation, the eyepiece position may be uncomfortable. Hence, EMs are implemented mostly for photography where accurate tracking is essential.

German equatorial mount. The German equatorial mount (GEM) is a type of equatorial mount having a T-shaped structure. The mount was invented by Joseph von Fraunhofer for the Great Dorpat Refractor that was built in 1824 [1].

The GEM has a major axis of rotation – the right ascension axis or polar axis – and a secondary axis of rotation – the declination axis [2–4]. Some German equatorial mounts lack the second axis. The pivot point for the right ascension axis of rotation is fixed on a tripod or a pier and is stationary. The right ascension axis is oriented to be parallel to the Earth's rotation axis. Around the polar axis, at the pivot base, on bearings, the right ascension shaft is attached and is coupled with the right ascension bar (arm) that rotates within 180 degrees or less. This bar has the major payload attached to one of its ends and a counterweight – to the other. The major payload consists of the pivot for the declination axis of rotation, again designed around a shaft on bearings. The declination shaft homes the base plate to which the telescope is fixed. If the telescope is not inherently balanced in respect to the declination shaft, then a second counterweight is used to balance the declination axis payload – the telescope.

The counterweight or the counterweights are the most distinctive features of the German equatorial mount. The rotation of the polar axis needs to have the same rate of rotation as the Earth's. By keeping the declination axis not moving, the mount will point the telescope at the same celestial object for at most 12 hours. The advantages of the German equatorial mount over the other most used equatorial mount, the fork equatorial mount, are stability, versatility, weight, and less vibrations. The major disadvantage of a German equatorial mount is its very high cost.

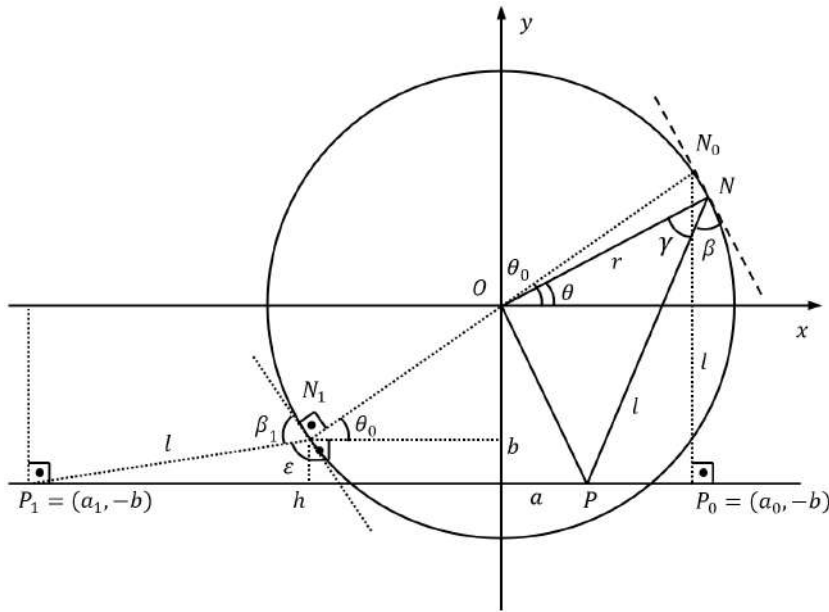


Fig. 1. Offset piston crank mechanism

The offset slider crank mechanism. The mechanism rotating the right ascension shaft of the GEM may be of various types. We chose an offset slider crank mechanism, well-known for its use in internal combustion engines [5,6], due to the availability of parts to build one and the low cost thereof. The task is formulated below. Initially, we solved the problem by numerical methods. Then we reached an analytical solution disclosed in the current article. The offset piston/slider crank mechanism is presented in Fig. 1.

Elements defining the offset piston crank mechanism (see Fig. 1):

- $\vec{O}$ – crank centre. This point is stationary.
- $\vec{N}$ – crank pin vector. This point travels on the unit circle with centre $\vec{O}$. $|\vec{N}|$ – crank radius. During the mechanism motion, vector $\vec{N}$ transitions monotonically from point $\vec{N}_0$ to point $\vec{N}_1$.
- θ – crank angle.
- $\vec{P}$ – piston pin vector. Piston pin starts its linear motion from point $\vec{P}_0 = (a_0, -b)$ and stops at point $\vec{P}_1 = (a_1, -b)$ along the pin path $\vec{P}_0\vec{P}_1$. For simplicity, $\vec{P}_0\vec{P}_1$ is chosen to be parallel to Ox axis.
- a – piston pin position along piston linear motion path $\vec{P}_0\vec{P}_1$.
- b – crank offset. The distance between $\vec{O}$ and piston linear motion path $\vec{P}_0\vec{P}_1$.
- $\vec{PN}$ – connecting rod. $|\vec{PN}| = l$ – connecting rod length.
- γ – angle between crank radius and connecting rod.

- $\beta = \frac{\pi}{2} - \gamma$ – angle between connecting rod and crank pin tangent to unity circle.

Conditions of the problem:

1. $\Delta\theta = \pi \cdot \theta$ decreases monotonically from start value θ_0 to end value θ_1 with piston pin travel from $\vec{P_0}$ to $\vec{P_1}$.
2. $b > 0$, $b \leq 1$ – offset parameter variable. The condition $b \leq 1$ is chosen for compactness of the mechanism and reasonable maximum ratio between offset and radius and connecting rod length and radius.
3. $l > b$ – parameter variable. If $l \leq b$, then $\Delta\theta < \pi$.
4. $a > 0$ – this variable is chosen so as to have $\vec{P_0N_0} \perp \vec{P_0P_1}$.

Problem. Let $\beta_{\max}$ be the maximum of $|\beta|$ in respect to a :

$$\max_a |\beta| = \beta_{\max}.$$

The problem the authors sought to resolve is to find the minimum in respect to $[b, l]$ of $\beta_{\max}$:

$$\min_{b,l} \left(\max_a |\beta| \right) = \min_{b,l} (\beta_{\max}).$$

Solution. After choosing $[b, l]$, it follows from θ monotonicity condition and $l > b$ condition that $\vec{P_0N}$ has to be perpendicular to $\vec{P_0P_1}$:

$$l - b = q \in (0..1) \Rightarrow l = b + q.$$

We define:

$$h = 2b - l = b - q.$$

For a_0 and a_1 we have:

$$\begin{aligned} a_0 &= \sqrt{1 - (l - b)^2} = \sqrt{1 - q^2} \\ a_1 &= -a_0 - \sqrt{l^2 - h^2}. \end{aligned}$$

Now we can express the starting crank angle θ_0 :

$$\sin \theta_0 = q; \quad \cos \theta_0 = a_0 = \sqrt{1 - q^2}.$$

Then we express angle ε related to the final crank pin position:

$$\cos \varepsilon = \frac{h}{l} = \frac{b - q}{b + q}; \quad \sin \varepsilon = \frac{2}{b + q} \sqrt{bq}.$$

For angle β from triangle OPN we have:

$$a^2 + b^2 = r^2 + l^2 - 2rl \cos \gamma = 1 + l^2 - 2l \cos \gamma$$

$$\cos \gamma = \frac{1 + l^2 - a^2 - b^2}{2l} = \sin \left(\frac{\pi}{2} - \gamma \right) = \sin \beta$$

$$\beta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \Rightarrow \sin \beta \in (-1, 1).$$

For this interval $\sin \beta$ is monotonic. Hence $\max_a |\beta|$ and $\max_a |\sin \beta|$ are reached for the same a .

$$\sin \beta_{\max} = \left(\frac{1 + l^2 - a^2 - b^2}{2l} \right) \Big|_{a=0} = \frac{1 + l^2 - b^2}{2l} = \frac{q^2 + 2bq + 1}{2(b + q)} = f(q, b)$$

For angle β_1 (angle $\beta_1 = -\beta$ in the final crank position) we have:

$$\beta_1 = \pi - (\theta_0 + \varepsilon)$$

$$\sin \beta_1 = \sin (\theta_0 + \varepsilon) = \sin \theta_0 \cos \varepsilon + \cos \theta_0 \sin \varepsilon$$

$$\sin \beta_1 = q \frac{b - q}{b + q} + \frac{2}{b + q} \sqrt{bq} \sqrt{1 - q^2} = \frac{bq - q^2 + 2\sqrt{bq} \sqrt{1 - q^2}}{b + q} = g(q, b).$$

We define a new function $m(q, b)$:

$$m(q, b) = \max \{f(q, b), g(q, b)\}.$$

We need to find the minimum of $m(q, b)$. To achieve this goal we find the (q, b) plane projection of the intersection curve of the two surfaces $f(q, b)$ and $g(q, b)$ by solving the equation $f(q, b) = g(q, b)$ for b (see Fig. 2):

$$\frac{q^2 + 2bq + 1}{2(b + q)} = \frac{bq - q^2 + 2\sqrt{bq} \sqrt{1 - q^2}}{b + q} \Rightarrow b(q) = \frac{(3q^2 + 1)^2}{16q(1 - q^2)}.$$

Then we derive the intersection curve as a function of q by substituting $b(q)$ into $f(q, b)$:

$$f(q, b(q)) = i(q) = \frac{q^5 + 6q^3 + 9q}{-7q^4 + 22q^2 + 1}.$$

We need to take into account the domain of angle β :

$$\beta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \Rightarrow \frac{3}{2}\pi - \varepsilon > \theta_0 \wedge \frac{\pi}{2} - \varepsilon < \theta_0, \quad \theta_0 \in \left(0, \frac{\pi}{2} \right), \quad \varepsilon \in \left(0, \frac{\pi}{2} \right),$$

$$\frac{\pi}{2} - \varepsilon \in \left(0, \frac{\pi}{2} \right) \Rightarrow \sin \left(\frac{\pi}{2} - \varepsilon \right) < \sin \theta_0 \Rightarrow \cos \varepsilon < \sin \theta_0 \Rightarrow \frac{b - q}{b + q} < q \Rightarrow$$

$$b < q \frac{(1 + q)}{(1 - q)} \Rightarrow \frac{(3q^2 + 1)^2}{16q(1 - q^2)} < q \frac{1 + q}{1 - q} \Rightarrow \sqrt{5} - 2 < q < 1.$$

Another inequality that must be satisfied so that $\Delta\theta = \pi$ is:

$$l + b \geq 1 \Rightarrow 2b + q \geq 1 \Rightarrow b \geq \frac{1 - q}{2}.$$

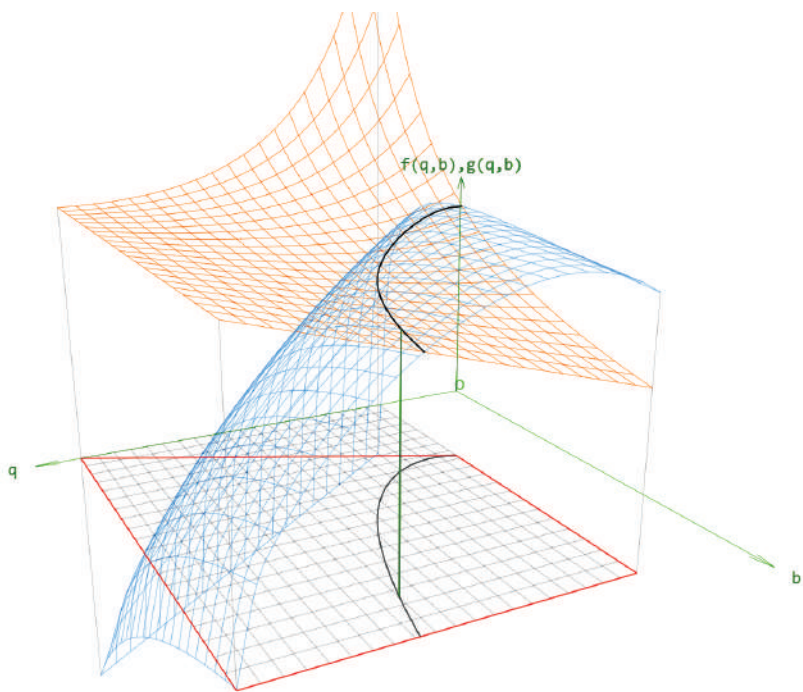


Fig. 2. The intersection of surfaces $f(q, b)$ and $g(q, b)$ and its projection in the Oqb plane (see text)

We calculate the minimum of $i(q)$:

$$\min_{\sqrt{5}-2 < q < 1} (i(q)) = \min_{\sqrt{5}-2 < q < 1} \left(\frac{q^5 + 6q^3 + 9q}{-7q^4 + 22q^2 + 1} \right) = \frac{9}{7} \sqrt{\frac{3}{7}} @ q = \sqrt{\frac{3}{7}}.$$

Finally, we use the fact that $m(q, b)$ has no local minima nor domain boundary minima lower than the minimum of $i(q)$ within the domain:

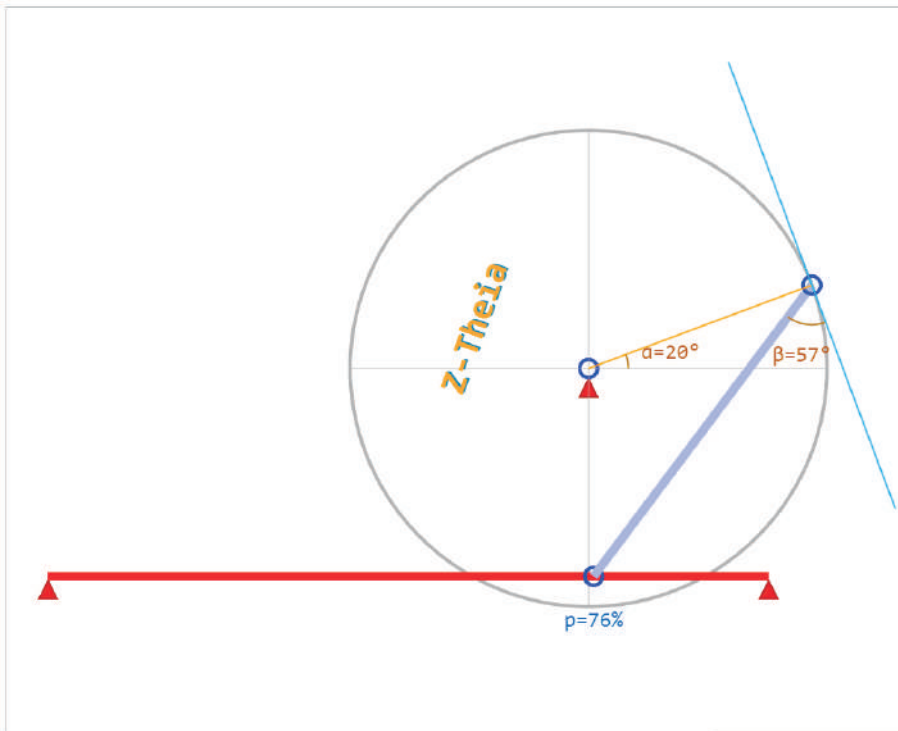
$$\sqrt{5} - 2 < q < 1, \quad \frac{1-q}{2} \leq b \leq 1.$$

The minimum $\beta_{\max}$ angle is:

$$\min_{0 < \beta_{\max} < \frac{\pi}{2}} (\beta_{\max}) = \arcsin \left(\frac{9}{7} \sqrt{\frac{3}{7}} \right) = 57.3198^\circ.$$

Then we calculate the offset of the mechanism:

$$b = \frac{(3q^2 + 1)^2}{16q(1 - q^2)} = \frac{4}{\sqrt{21}}.$$



Circle 1: $(x - x_1)^2 + (y - y_1)^2 = r_1^2$
 Circle 2: $(x - x_2)^2 + (y - y_2)^2 = r_2^2$
 Add these two equations:
 $-2x(x_1 - x_2) - 2y(y_1 - y_2) = (r_1^2 - r_2^2) - (x_1^2 - x_2^2) - (y_1^2 - y_2^2)$
 $x(x_1 - x_2) + y(y_1 - y_2) = -0.5((r_1^2 - r_2^2) - (x_1^2 - x_2^2) - (y_1^2 - y_2^2))$
 $y = -x(x_1 - x_2)/(y_1 - y_2) - 0.5((r_1^2 - r_2^2) - (x_1^2 - x_2^2) - (y_1^2 - y_2^2))/(y_1 - y_2)$
 $y = Ax + B$
 $A = -(x_1 - x_2)/(y_1 - y_2)$
 $B = -0.5((r_1^2 - r_2^2) - (x_1^2 - x_2^2) - (y_1^2 - y_2^2))/(y_1 - y_2)$
 Substitute y in equation for Circle 1:
 $x^2 - 2xx_1 + x_1^2 + A^2x^2 + 2Ax(B - y_1) + (B - y_1)^2 = r_1^2$
 $x^2(1 + A^2) + 2x(A(B - y_1) - x_1) + x_1^2 + (B - y_1)^2 - r_1^2 = 0$
 $a = 1 + A^2, k = A(B - y_1) - x_1, c = x_1^2 + (B - y_1)^2 - r_1^2$
 $x_{12} = (-k \pm \sqrt{k^2 - ac})/a$
 $y_{12} = Ax_{12} + B$

```
p      = 0.7570
alpha  = 20.4743° (rotation angle)
beta   = 57.3043° (tangent angle)
r      = 1.00
a      = 3.02
a0     = 0.76
a1     = 2.27
b      = 0.87
l      = 1.53
|beta|max = 57.3198°
```

Fig. 3. Offset piston crank mechanism simulation in JavaScript

And the cranking rod length:

$$l = b + q = \sqrt{\frac{7}{3}}.$$

The auxiliary element h is required to calculate the piston path coordinates:

$$h = 2b - l = b - q = \frac{1}{\sqrt{21}}.$$

And the last parameter of the mechanism is the piston path:

$$\begin{aligned} a_0 &= \sqrt{1 - q^2} = \frac{2}{\sqrt{7}} \\ a_1 &= -a_0 - \sqrt{l^2 - h^2} = -\frac{6}{\sqrt{7}} \\ a_0 - a_1 &= \frac{8}{\sqrt{7}}. \end{aligned}$$

Using this analytical solution we then simulated the mechanism in JavaScript (see Fig. 3).

Conclusion. The optimized offset slider crank mechanism provides accuracy, simplicity and low cost for the rotation mechanism of the right ascension axis of the German equatorial mount. By coupling the mechanism with feedback from electronic compass and accelerometer sensors, the onboard computer is capable of accurately positioning the mount payload. The so designed equatorial mount is suitable for solar observations [7] and photography and also for other astrophotography tasks.

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