

NUMERICAL INVESTIGATION OF IMPACT LOADS ON RC BEAMS

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Abstract

Reinforced concrete (rc) beams are significant elements which are designed to resist shear, bending and torsional forces. However, sudden impact loads whose effects may reach very high values in a short span of time are ignored in the design phase. So, rc beams with insufficient stirrups exhibit reduced shear capacity, making them vulnerable to impact loads, which generate high strain rates and rapid stress distribution. This study aims to investigate the impact effect on rc beams having two different stirrup configurations. As, carbon fibre reinforced polymer (cfrp) has been widely used to strengthen the structural elements in recent years, rc beams are strengthened by cfrp strips under impact loading. Non-linear incremental dynamic analysis is performed by Abaqus finite elements analysis software. Acceleration, displacement and impact load values are obtained for the first drop of the loading hammer from the software. These values are exhibited by time dependent graphs. In addition, fracture patterns of the rc beams are visually presented. The analysis outputs are evaluated, and suggestions are proposed in the end.

Key words: Abaqus, carbon fibre, impact, non-linear analysis

Introduction. Reinforced concrete (rc) beams play a critical role in the overall structural framework of buildings and other infrastructure. They are designed to carry and transfer loads from horizontal elements like floors, roofs, or slabs to vertical supports such as columns and walls. These beams are engineered to resist bending, shear, and torsion, ensuring the stability and safety of the entire

structure. Given their vital role in load distribution and overall structural performance, the design, detailing, and maintenance of rc beams are crucial to the safety and longevity of any construction. Their failure can compromise the entire structure, highlighting the importance of ensuring adequate shear and bending capacity, particularly under varying load conditions.

Carbon fibre reinforced polymer (cfrp) is a high-performance material widely used for strengthening rc beams, especially in cases where there is a lack of transverse reinforcement. When applied externally to rc beams, cfrp sheets significantly enhance the beam's shear capacity, preventing the propagation of cracks that typically occur due to insufficient stirrups. In cases where stirrup deficiencies compromise the beam's ability to resist shear forces, cfrp acts as an additional reinforcement, working in conjunction with the longitudinal steel bars. Cfrp effectively distributes the shear stresses and provides additional confinement to the concrete, improving the beam's overall strength and durability. This strengthening method is a cost-effective and efficient way to address the lack of stirrups and extend the service life of rc beams, ensuring they perform safely under both static and dynamic loads.

Impact loads are sudden, high-intensity forces that can cause immediate and significant damage to structural elements. These loads are typically applied in a short time span, leading to a quick transfer of energy, which can overwhelm a structure's capacity to resist deformation or failure. Examples of sudden impact loads include vehicle collisions with buildings or bridges, falling rocks, explosions, and the forces exerted during seismic events. The effects of these impact loads are often more severe than static loads because of the dynamic nature of the forces involved. Structural elements like beams, columns, and slabs may experience rapid acceleration, displacement, and even fracture due to the shockwaves generated by impact. These loads can lead to localized failure or, in extreme cases, progressive collapse if the structure cannot redistribute the forces effectively. Proper consideration of impact loading is crucial in the design and retrofitting of structures.

Experimental studies investigating the effects of impact loads on structural elements evaluate the behaviour of materials and specimens [1, 2]. However, designing test setups to simulate impact loads on structural elements can be a complex and costly process. The required equipment, such as accelerometer, lvdt, load sensor can be expensive, often placing a significant financial burden on the studies. So, many researchers turn to numerical analysis using well-known software, to simulate the behaviour of structural elements under impact loading [3, 4]. By using simulation tools of the software, researchers can efficiently model the impact response of structural elements, obtaining valuable insights without the need for costly physical tests. These tools also allow for the exploration of multiple scenarios, different material properties, and varying loading conditions, offering cost-effective alternative to experimental testing. Moreover, these simulations could be used in the validation of experimental results, ensuring that the numerical models

provide accurate predictions [5–7].

In this study, the behaviour of rc beams with different stirrup and cfrp configuration is numerically investigated under impact loading. Abaqus which is widely used in the simulations to study the impact response of structural elements is utilized in the analysis [8]. The advanced finite element analysis tools allow researchers to accurately model complex interactions, material behaviours, and failure mechanisms under impact loads. Abaqus provides a reliable platform for simulating various impact scenarios, enabling detailed analysis of stress distribution, crack propagation, and the overall structural response without the need for extensive physical testing. So, it is thought that the analysis results of this study will contribute to the literature about multiple impact effects on structural elements.

Properties of the rc beams. Before performing non-linear incremental dynamic analysis, three-dimensional finite element models of the simulated rc beams are created in the software. Section dimensions are 250×450 mm for each specimen. Besides, span length of the beams is 2000 mm. While diameter of the longitudinal reinforcement is $\varnothing 10$ mm, the value is taken to be $\varnothing 6$ mm for the stirrups. All specimens have three longitudinal tension and two compression reinforcements at bottom and top side, respectively. On the other hand, two different values are considered between the stirrups such as 200 and 300 mm. While reinforcement details are given in Fig. 1a, cfrp strips are shown in Fig. 1b for Specimen 2.

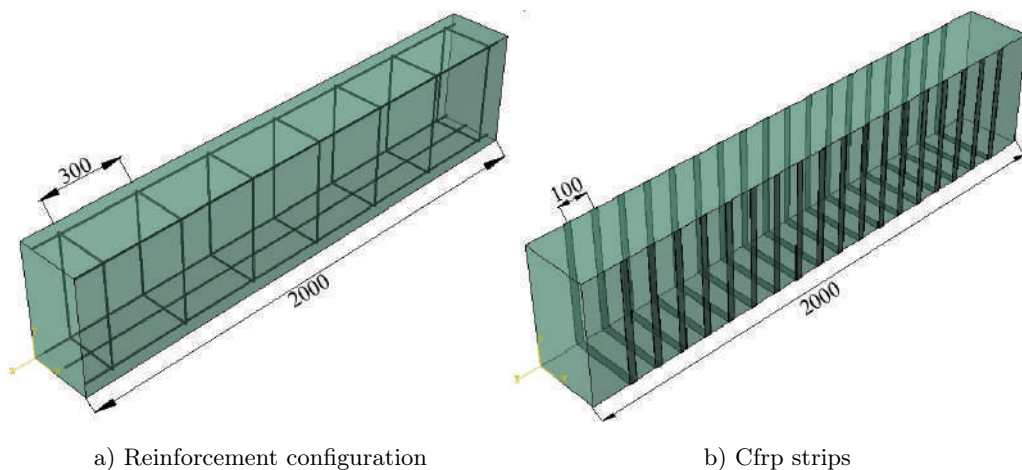


Fig. 1. Details of Specimen 2

The other examined parameter of this study is the cfrp strips. The specimens are externally wrapped by cfrp strips with a constant width of 25 mm with the spacing of 100 and 200 mm, respectively. Cfrp strips are applied to the three sides of the rc beams with the shape of U. A perfect adherence is utilized between the rc

beams and cfrp in the numerical analysis. Reinforcement and cfrp configuration of Specimen 2 is shown in Fig. 1. While cfrp strips are placed with a distance of 100 mm for Specimens 1 and 2, the value is considered 200 mm for Specimens 3 and 4. On the other hand, while the spacing value for stirrups are 200 mm for Specimens 1 and 3, the spacing value is taken to be 300 mm for Specimens 2 and 4.

Numerical analysis. Explicit module of the software that yields the most reliable outputs in incremental dynamic problems such as blast and impact is used in the numerical analysis. This module successfully performs numerical simulations involving high-speed collisions, contact, and large deformations. Besides, many failure characteristics and advanced material models can be found in the library of Abaqus/Explicit. Recent studies demonstrate that researchers have been using Explicit module to simulate the interaction of collided parts in impact analysis.

The finite elements model consists of the concrete beam, steel reinforcement, cfrp strip, loading plate, support, and the loading hammer. So, three dimensional models have been modelled in the first place. For this purpose, three dimensional, 8-node linear brick, hexahedron (C3D8R) element type which is proper for dynamic simulations is utilized in the software. In addition, the steel rebars are modelled by two node beam element type (B31). The interaction between the concrete slab and steel reinforcement is defined using the embedded region constraint feature of the software.

After creating the three-dimensional models, material properties are assigned to related sections in the next step. For this purpose, the material properties of concrete, steel reinforcement, impact hammer, and loading plate are defined in the software. The concrete damage plasticity (cdp) model, which is a continuum, plasticity-based, damage model for concrete type materials is considered to define the non-linearity of concrete. This model exists in the material library of the software and enables users to characterize the uniaxial tensile and compressive response of concrete separately.

In cdp model, MANDER's stress-strain model for unconfined concrete is considered to define the behaviour of concrete in compression [9]. In the compression region, a linear path is followed until the value of initial yield (σ_{c0}). Beyond this point, the plastic response consists of stress hardening until reaching the ultimate stress (σ_{cu}), followed by strain softening, which represents the progressive deterioration of concrete. In the tension region, the material exhibits linear elastic behaviour until the failure stress (σ_{t0}), corresponding to the initiation of the first microcrack. After the failure stress, a softening stress-strain response governs the propagation of microcracks, leading to a gradual reduction in stiffness.

Two key damage variables, d_t and d_c , are used to degrade the elastic stiffness. These variables range between $0 < d_t, d_c < 1$, where 0 represents an undamaged material, and 1 indicates a complete loss of strength. The stress-strain response in

both tension and compression can be defined using specific equations, such as Eqs. (1, 2). In these equations, the parameters in the following equations, E_0 , $\tilde{\varepsilon}_c$, and $\tilde{\varepsilon}_t$ represent the initial elastic modulus of the material, the equivalent plastic strain in compression, and the equivalent plastic strain in tension, respectively. These parameters are essential in defining the stress-strain behaviour and the evolution of damage in both compression and tension within the cdp model.

$$(1) \quad \sigma_c = (1 - d_c)E_0(\varepsilon_c - \tilde{\varepsilon}_c^{pl})$$

$$(2) \quad \sigma_t = (1 - d_t)E_0(\varepsilon_t - \tilde{\varepsilon}_t^{pl}).$$

Eqs. (1) and (2) are rearranged to express the damage variables d_c and d_t more clearly, as shown in Eqs. (3, 4). The elastic modulus is denoted by E_0 in these equations. Additionally, the equivalent plastic strains in compression and tension are $\tilde{\varepsilon}_t^{pl} = b_t \left(\varepsilon_t - \frac{\sigma_t}{E_0} \right)$ and $\tilde{\varepsilon}_c^{pl} = b_c \left(\varepsilon_t - \frac{\sigma_c}{E_0} \right)$, respectively. Based on the recommendations of researchers, the values of b_c and b_t are set to 0.7 and 0.1 in the numerical analysis [10].

$$(3) \quad d_c = 1 - \frac{\sigma_c/E_0}{\tilde{\varepsilon}_c^{pl} \left(\frac{1}{b_c} - 1 \right) + \sigma_c/E_0}$$

$$(4) \quad d_t = 1 - \frac{\sigma_t/E_0}{\tilde{\varepsilon}_t^{pl} \left(\frac{1}{b_t} - 1 \right) + \sigma_t/E_0}.$$

The cdp model is defined by five key parameters which are also utilized in concrete modelling. These parameters, which characterize the yield surface function, include the dilation angle (ψ), the flow potential eccentricity (e), the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress (σ_{b0}/σ_{c0}), the coefficient that determines the shape of the deviatoric cross-section (K_c), and the viscosity parameter (μ). These values are defined as 30, 0.10, 1.16, 0.6667, 0.0001, respectively. The ultimate concrete strain value (ε_{cu}) is taken as 0.003 in the software. Besides, the ultimate strain in tension (ε_{tu}) is defined as $\varepsilon_{tu} = 10\varepsilon_{cr}$, where (ε_{cr}) represents the strain at ultimate tensile strength. The tensile behaviour is assumed to increase linearly up to ε_{cr} , and then decrease until (ε_{tu}). The elastic modulus of concrete (E_c) is calculated based on the compressive strength (f_c) using Eq. (5). Additionally, the tensile strength of concrete (f_t) is determined by Eq. (6) [11, 12]. Other parameters such as Poisson's ratio and density are taken as 0.2 and 2400 kg/m³. Since low concrete strength is typically associated with cases of insufficient stirrup usage, the compressive strength of concrete is considered 12 MPa.

$$(5) \quad E_c = 4700\sqrt{f_c}$$

$$(6) \quad f_t = 0.623\sqrt{f_c}.$$

In the next step of the material definition, properties of the steel reinforcement and the impact hammer are defined in the software. Both longitudinal and transverse reinforcements are embedded in the beams and tie contact property of the software is utilized to provide the adherence between the beam and the reinforcement. The steel reinforcement is modelled as an elastic-perfectly plastic material, while the hammer is represented using a linear elastic model. The values of Poisson's ratio, density, elastic modulus, shear modulus, bulk modulus and yield strength for the steel material are defined as 0.30, 7850 kg/m³, 200 000 MPa, 76 923 MPa, 166 670 MPa, and 420 MPa, respectively.

As cfrp strengthening is expected to improve the performance of rc elements with insufficient shear strength as well as the impact resistance, cfrp strips are applied to the specific parts of the rc beams with two different spacings. The material properties of the cfrp are taken from the manufacturer whose thickness, elastic modulus, tensile strength and ultimate tensile strain are 0.12 mm, 4100 MPa, 231 000 MPa, and 1.7%, respectively.

The interface between cfrp and concrete is represented by a cohesive zone model that follows a bilinear traction-separation law, characterized by effective traction τ , and effective opening displacement δ . In this figure, stiffness, the local strength of the material, opening displacement corresponding to fracture, and the energy required for opening crack are used in the definition of τ – δ relationship and symbolized by K_0 , $\tau_{\max}$, δ_f , and G_{cr} , respectively. Concrete and cfrp material models are presented in Fig. 2.

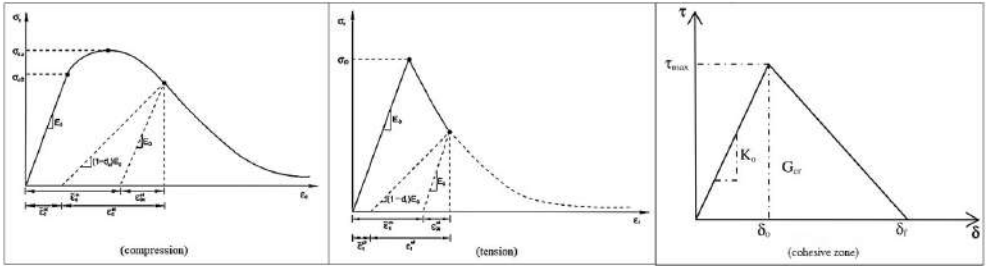


Fig. 2. Material models

K_0 is defined according to Eq. 7. While t_i and t_c are epoxy thickness and concrete thickness, G_i and G_c are the shear modulus of epoxy and concrete, respectively. So, the value of K_0 is calculated as 640 MPa. The cohesive zone modelling parameters are utilized to provide the interface between concrete and cfrp material. As no debonding is considered between the surfaces of the materials,

the value of $\tau_{\max}$ is taken as 3 MPa as suggested in the literature [10].

$$(7) \quad K_0 = \frac{1}{\frac{t_i}{G_i} + \frac{t_c}{G_c}}.$$

Mesh convergence trials have been conducted for Specimen 1 between 10 mm and 35 mm finite element sizes, since more accurate results are obtained when finite element models are separated into smaller parts. It is concluded that there are no big differences among the results around the mesh size of 20 mm. However, the solution time extends for the values smaller than 15 mm. Based on these findings, mesh size is utilized as 15 mm for all geometries in the software.

Results. The software employs contact algorithms to simulate the interaction between colliding components. During impact loading, the surface of the hammer is defined as master, while the specimen's surface is designated as slave. A surface-to-surface contact property is assigned between these surfaces. Additionally, both tangential and normal contact properties are considered to accurately represent the interaction in the analysis. The hammer is restricted to move only in the vertical direction, ensuring that gravity is the only external force acting on it. Besides, support regions of the rc beams with boundary conditions in horizontal, vertical, and axial directions are defined for both ends of the specimens in the software.

Time increments and steps have significant effects on the analysis results. As the investigated problem is an incremental dynamic one, the numerical analysis is performed for small time increments in the software. So, the analysis is repeated several times according to time increments. While 0.060 sec of time increments have been used before the contact between the hammer and the specimen, time increments are defined as 2×10^{-8} s from the beginning of the contact.

In the next stage of the numerical analysis, a study has been carried out to define the strain-rate effects on the material models. As the mechanical behaviour of rc structural elements under sudden dynamic loads differs significantly from their behaviour under quasi-static loads, neglecting strain-rate effects may lead to deviations from accurate analysis results. Therefore, accounting for the strain rate effect is essential for precisely evaluating the dynamic responses of specimens in the software.

The dynamic increase factor (dif) defining the ratio of dynamic strength to static strength concerning strain rate is utilized to represent the effect of strain-rate effects in concrete and steel materials. For this purpose, empirical equations have been proposed in the literature [13]. By applying these equations to the material models in the software, the stress-strain curves are adjusted for specific strain rates using an iterative solution algorithm that incorporates the dif, as described in the literature [14].

Concrete material is assessed by including strain rate effects through the ratio

of dynamic strength to static strength. The compressive and tensile dif equations for concrete have been applied in numerical models [15]. While *cdif* symbolizes the compressive dif, *tdif* represents the tension dif in Eqs. (8,9). In these equations, f_{cd} and f_{td} are dynamic compressive and tensile strengths at the strain rate, $\dot{\epsilon}_d$. Additionally, f_{cs} and f_{ts} are static compressive and tensile strengths, respectively.

$$(8) \quad CDIF = \frac{f_{cd}}{f_{cs}} = \left\{ \begin{array}{ll} 0.049(\log \dot{\epsilon}_d) + 1.2165 & \text{for } (\dot{\epsilon}_d \leq 30s^{-1}) \\ 0.8988(\log \dot{\epsilon}_d)^2 - 2.8255(\log \dot{\epsilon}_d) + 3.4907 & \text{for } (\dot{\epsilon}_d > 30s^{-1}) \end{array} \right\}$$

$$(9) \quad TDIF = \frac{f_{td}}{f_{ts}} = \left\{ \begin{array}{ll} 0.26(\log \dot{\epsilon}_d) + 2.06 & \text{for } (\dot{\epsilon}_d \leq 1s^{-1}) \\ 2(\log \dot{\epsilon}_d) + 2.06 & \text{for } (1s^{-1} < \dot{\epsilon}_d \leq 2s^{-1}) \\ 1.44331(\log \dot{\epsilon}_d) + 2.2276 & \text{for } (2s^{-1} < \dot{\epsilon}_d \leq 150s^{-1}) \end{array} \right\}$$

In addition to the equations above, Eq. (10) presents the dif value of the steel material for a given strain rate, $\dot{\epsilon}$. The α coefficient in this equation could be expressed through f_y that is the yield strength of steel material in MPa as presented in Eq. (11).

$$(10) \quad DIF = \left(\frac{\dot{\epsilon}}{10^{-4}} \right)^\alpha$$

$$(11) \quad \alpha = 0.074 - \frac{0.04f_y}{414}.$$

Before performing numerical analysis of rc beams, validation of the finite elements model has been generated through an experimental study of a reference concrete beam whose section sizes are 100×200 mm. The reference specimen has been subjected to impact loading by a 12 kg hammer that is dropped from a height of 1000 mm. Accelerations have been measured from specific parts of the concrete beam. In addition, a dynamic load cell is placed in the edge part of the hammer to obtain the impact load values. Experimental and numerical results of the maximum accelerations and impact loads are given in Table 1a. It is seen that a good relationship is established between both results. However, there are slight differences between experimental and numerical values. It is thought that inner cracks of concrete, friction effects and slight support movements during impact experiments are the main reasons of the error rates.

After validating the proposed numerical model, tc beams have been analyzed by considering dif coefficients that represent the change in strength values of the materials owing to strain rates under the same level of impact energy. While the mass of the hammer is 80 kg, drop height is taken to be 1800 mm. Minimum and maximum acceleration, maximum displacement and impact load values are determined from the software for each specimen. The values are presented in

T a b l e 1
Analysis results

a) Comparison of reference specimen

Specimen	Experimental acceleration (m/s ²)		Numerical acceleration (m/s ²)		Experimental impact load (kN)	Numerical impact load (kN)
Reference	-2668	2915	2517	2749	17.13	18.56

b) Specimens 1–4

Specimen	Stirrup spacing (mm)	Cfrp spacing (mm)	Acceleration (m/s ²)		Displacement (mm)	Impact load (kN)
Specimen 1	200	100	-7014	6683	19.64	33.83
Specimen 2	300	100	-6329	6036	25.82	32.64
Specimen 3	200	200	-6227	6407	23.21	33.05
Specimen 4	300	200	-5351	5845	32.17	31.76

Table 1b. For better understanding, the values for Specimen 4 are presented as time-dependent graphs in Fig. 3a.

In the last step of the numerical analysis, the crack patterns of the concrete and rc beams are obtained through damage function of Abaqus. The fracture distributions could be visually followed by this function. It is seen that fractures concentrate on the mid-point of the specimens where impact loading has been implemented. Also, damages become more intense on Specimen 4 having the biggest distance between stirrups and cfrp strips. While the crack propagation of the concrete beam is shown in Fig. 3b, the damage distributions of the rc specimens are exhibited in Fig. 3c.

Conclusions. Various factors may necessitate the strengthening of rc beams, including increased service loads, material deterioration over time, design deficiencies in older structures, and damage from environmental exposure or earthquakes. Effective strengthening techniques, such as cfrp bonding help enhance the rc beam's capacity, prolonging the service life of the structure while minimizing demolition and reconstruction costs. Moreover, cfrp applications are more efficient compared to traditional strengthening methods. As research and development in cfrp technology continues, its implementation is expected to expand, offering sustainable solutions for extending the service life of rc structural elements.

In this study, the validation of the proposed finite elements model has been carried out based on the results of experimental impact loading applied to a reference concrete beam in the first place. After obtaining a good correlation between the experimental and numerical analysis results, four different rc beams having different stirrup and cfrp configuration have been evaluated under sudden impact loads. For this purpose, non-linear incremental dynamic analysis under constant

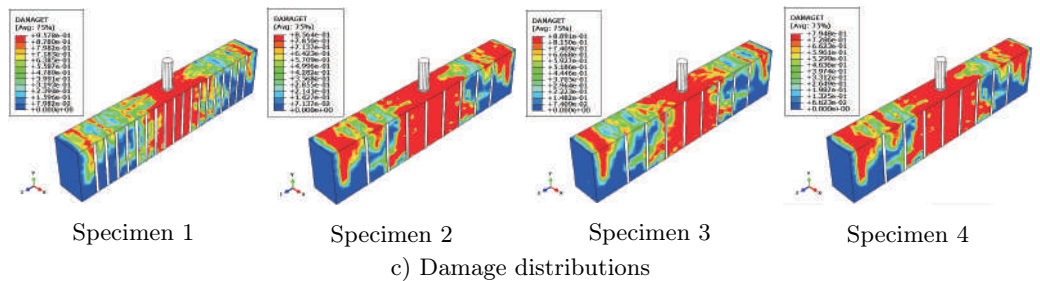
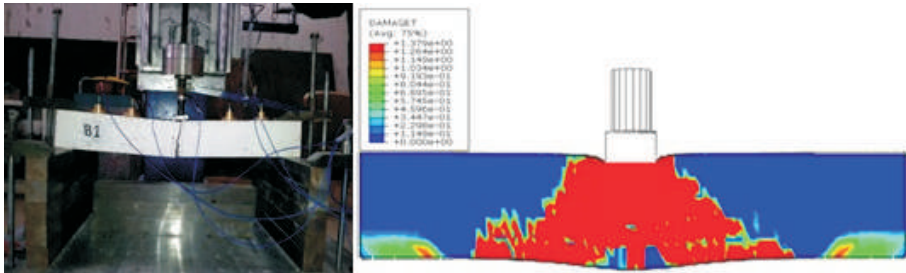
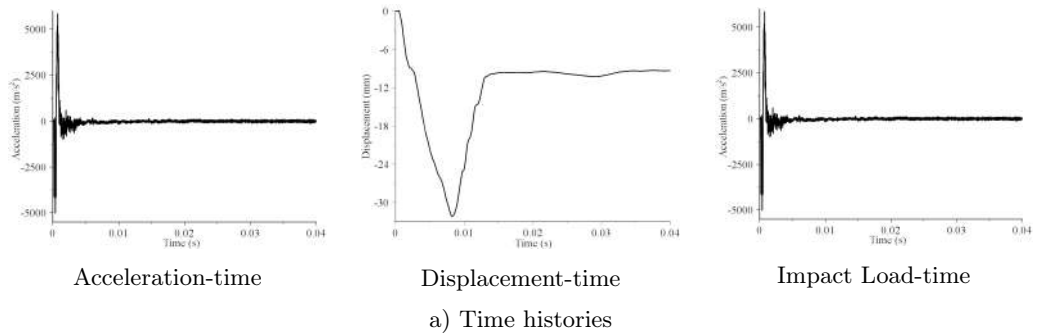


Fig. 3. Analysis results

level of impact energy is performed by Abaqus software that is commonly used for such solutions. As distinct from many studies in the literature, strain-rate effects are considered to achieve the behaviour of the specimens properly. Acceleration, displacement, and impact load values are obtained for each rc beam after numerical analysis. In addition, fracture propagation is presented under impact load for each stirrup and cfrp configuration.

The analysis results reveal that cfrp strengthening has significantly improved the impact behaviour of rc beams. The highest acceleration and the lowest displacement values are obtained for Specimen 1 having the highest stiffness with the smaller spacing stirrups and cfrp strips. While the acceleration values decrease, displacement values increase as the spacing values between the stirrups and cfrp

strips increase. However, there are no big differences between impact load values of the specimens. Constant value of drop height and mass of the impact hammer is the main reason for this situation.

The fracture patterns are also obtained for the reference concrete beam and rc beams in the software. It is seen that maximum damage accumulates around impact point for each case. Besides, the effect of spacings between stirrups and cfrp strips is visually exhibited. Maximum damage is observed in Specimen 4 which has the biggest spacing values. On the other hand, minimum damage has occurred in Specimen 1 having the highest rigidity among all specimens. Finally, the finite element models of this study can give ideas to researchers in the design of rc beams with cfrp strips. It is evaluated that this study will contribute to the literature by considering strain-rate effects under impact loading. Thus, the labour and cost spent in experimental studies will be saved when accurate modelling and analysis conditions are provided.

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