

MARKOV BRANCHING PROCESS WITH INFINITE
VARIANCE AND POISSON IMMIGRATION WITH
DECREASING INTENSITY

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Abstract

We consider critical Markov branching processes with infinite variance of the offspring distribution which admit an immigration at the jump-points of a non-homogeneous Poisson process, assuming that the mean number of immigrants is infinite and the intensity of the Poisson process converges to zero. The probability for non-hitting zero is asymptotically investigated and limiting distributions are obtained, under suitable normalization.

Key words: Markov branching process, limit theorems, infinite variance, non-homogeneous immigration

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1. Introduction. The paper studies Markov branching process $\{Y(t), t \geq 0\}$ with immigration in time-moments generated by a Poisson measure with a local intensity $r(t)$. We consider the critical case when the offspring distribution belongs to a domain of attraction of a stable law with mean one and an infinite variance. The distribution of immigrants belongs to the domain of attraction of a stable law with infinite mean and $r(t)$ is a regularly varying function (r.v.f.) converging to zero. Note that the investigated branching processes are non-homogeneous in time. The present work in fact continues the investigations of this model introduced in [1], where $r(t)$ converges to infinity. The model when $r(t)$ converges

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to a positive constant, is considered in [2]. The case studied in the present paper is more complicated and the obtained results are quite different.

The first model of branching process with immigration was formulated by SEVASTYANOV [3] in the case of a time homogeneous Poisson process. Branching processes with time non-homogeneous immigration were first proposed by DURHAM [4] and FOSTER and WILLIAMSON [5]. Further results can be found in a review paper of RAHIMOV [6]. Critical Sevastyanov age-dependent branching processes with non-homogeneous Poisson immigration were studied in [7] and the case of critical multitype Markov branching processes was considered by MITOV et al. [8]. We have to point out that the results in all these models were obtained in the case of finite first and second offspring characteristics as well as those of the immigration components.

ZOLOTAREV [9] was the first who considered Markov branching processes with infinite offspring variance. Note that the asymptotic behaviour of branching processes is quite different in the case of finite or infinite offspring variance. PAKES [10] investigated critical Markov branching processes with infinite offspring variance and immigration with a finite mean occurring at time-points of a homogeneous Poisson process. SAGITOV [11] studied multi-type Markov branching processes in the case of homogeneous Poisson immigration with infinite second moments of the offspring distributions and infinite means of the immigrants. Branching processes with time non-homogeneous immigration find applications for investigating the dynamics of biological systems, particularly cellular populations (see, for example, [12]). In these applications, the stem cells are often considered an immigration component.

The description of the considered model and the basic equations are presented in Section 2. The asymptotic behaviour of the $P_t = \mathbf{P}\{Y(t) > 0\}$ (probability for non-hitting zero state at moment t) is given in Section 3 (Theorems 3.1 and 3.2). The conditions under which P_t converges to zero as $t \rightarrow \infty$ with various rates depending on the parameters of reproduction (γ), of immigration (α) and of the Poisson measure (θ) are presented. Under the conditions of Theorem 3.2(ii) P_t converges to a positive probability less than 1 which is exactly calculated. Finally, at the conditions of Theorem 3.2(iii) we obtain that $P_t \rightarrow 1$ as $t \rightarrow \infty$.

Limiting distributions are presented in five theorems given in Section 4. In fact we obtained eight different types of limiting distributions. In Theorems 4.1(i), 4.2(ii), and 4.4(ii) under suitable additional conditions we prove a stationary discrete time limiting distribution and we obtain an integral form for the corresponding probability generating function (p.g.f.) depending only on the local characteristics of the process (offspring p.g.f. $f(s)$ and the p.g.f. of the immigrants $g(s)$, see Comment 4.1). For the other limiting distributions under a normalization with a suitable r.v.f. we obtain the corresponding Laplace transforms. The most interesting result is presented in Theorem 4.1(iii), where we have two mutually singular limiting distributions, where the first one is an improper discrete distribution with

an atom at infinity and the second one under a suitable normalization has an atom at zero. Finally in Theorem 4.5(iii) we obtain non conditional limiting distribution under a suitable normalizing r.v.f. and the limiting random variable is stable with parameter α .

2. Model and equations. The classical Markov branching process $\{Z(t)\}$ is defined by the live-time distribution function (d.f.) $G(x) = \mathbf{P}\{\tau \leq x\} = 1 - e^{-\mu x}$, $x \geq 0$, $\mu > 0$, and by the offspring probability generating function (p.g.f.) $f(s) = \mathbf{E}[s^\xi]$. Then $F(s; t) = \mathbf{E}[s^{Z(t)} | Z(0) = 1]$, $t \geq 0$, $s \in [0, 1]$, is a unique solution of the non-linear equation (see e.g. [13])

$$\frac{\partial F(s; t)}{\partial t} = \mu[f(F(s; t)) - F(s; t)].$$

Consider the process (S_k, I_k) , $k = 0, 1, 2, \dots$, independent of $\{Z(t) : t \geq 0\}$, where $0 = S_0 < S_1 < S_2 < \dots$ are jump points of a non-homogeneous Poisson process $\nu(t)$ and the random variables $\{I_k\}$ are independent, identically distributed (i.i.d.) with a p.g.f. $g(s) = \mathbf{E}[s^{I_1}]$. Let $r(t)$ be the intensity of $\nu(t)$ with the mean measure $R(t) = \int_0^t r(u) du$.

At every jump-point S_k , a random number I_k of new particles immigrate into the process and they participate in the evolution as the other particles. Define this new process as follows

$$Y(t) = \sum_{k=1}^{\nu(t)} \sum_{j=1}^{I_k} Z^{(k,j)}(t - S_k), \quad t \geq 0, \quad Y(0) = 0,$$

where $\{Z^{(k,j)}(t) : t \geq 0\}$ are i.i.d. copies of $\{Z(t) : t \geq 0\}$. The process $\{Y(t) : t \geq 0\}$ is called a Markov branching process with non-homogeneous Poisson immigration (see [1] and [2]).

For $\Phi(s; t) = \mathbf{E}[s^{Y(t)} | Y(0) = 0]$ we have the following presentation

$$\Phi(s; t) = \exp \left\{ - \int_0^t r(t-u)(1 - g(F(s; u))) du \right\}, \quad \Phi(0; t) = 1.$$

The proof can be seen in [12] and for the more general multitype case in [8].

From now on we assume for the intensity of the Poisson process, that

$$r(t) = t^{-\theta} L_R(t), \quad \text{where } \theta > 0,$$

and $L_R(\cdot)$ is a slowly varying function (s.v.f.) at infinity. Note that the case $\theta < 0$ is investigated in [1] and the case when $r(t)$ converges to a constant is considered in [2].

Further on we suppose that the offspring p.g.f. $f(s)$ admits the representation

$$f(s) = s + (1-s)^{\gamma+1} L \left(\frac{1}{1-s} \right), \quad s \in [0, 1),$$

where $\gamma \in (0, 1]$ and $L(\cdot)$ is s.v.f. at infinity. It means that the process $\{Z(t), t \geq 0\}$, is critical, i.e. $\mathbf{E}\xi = f'(1) = 1$. If $\gamma < 1$ the offspring variance is infinite. If $\gamma = 1$ the offspring variance can also be infinite, depending on the properties of the s.v.f. $L(\cdot)$. The offspring variance is finite when $L(t) \rightarrow b < \infty$. Then the process will be a particular case of the multitype Markov processes with non-homogeneous Poisson immigration investigated in [8].

For the p.g.f. of the immigrants we will suppose from now on that

$$g(s) = 1 - (1 - s)^{\alpha l} \left(\frac{1}{1 - s} \right), \quad s \in [0, 1],$$

where $\alpha \in (0, 1]$ and $l(x)$ is a s.v.f. at infinity. If $\alpha = 1$ and $l(x) \rightarrow m \in (0, \infty)$, then $\mathbf{E}[I_k] = m$ is finite. If $\alpha \in (0, 1)$, the mean number of immigrants is infinite. Note that in the case when $\alpha = 1$ the mean number of immigrants can be infinite or finite depending on the s.v.f. $l(\cdot)$.

3. Probability for non-hitting zero state. Let $q(t; s) = 1 - g(F(s; t))$, $q(t) = q(t; 0)$, $Q(t) = \int_0^t q(u) du$ and $Q = \int_0^\infty q(u) du$.

Introduce $P_t = \mathbf{P}\{Y(t) > 0\} = 1 - \Phi(0; t)$.

Theorem 3.1. Let $t \rightarrow \infty$.

- (i) If $\theta \geq 1$ and $\alpha/\gamma \geq 1$, then $P_t \sim R(t)q(t) + Q(t)r(t)$.
- (ii) If $\theta \geq 1$ and $0 < \alpha/\gamma < 1$, then $P_t \sim R(t)q(t)$.
- (iii) If $0 < \theta < 1$ and $\alpha/\gamma \geq 1$, then $P_t \sim r(t)Q(t)$.

Comment 3.1.

- (i) If $\theta > \alpha/\gamma \geq 1$, then $R < \infty$ and $P_t \sim RL_Q(t)t^{-\alpha/\gamma}$. If $\alpha/\gamma > \theta \geq 1$, then $Q < \infty$ and $P_t \sim QL_R(t)t^{-\theta}$. Note that $L_Q(t)$ and $L_R(t)$ are s.v.f.
- (ii) If $\theta = \alpha/\gamma > 1$, then $P_t \sim [RL_Q(t) + QL_R(t)]t^{-\theta}$. If $\theta = \alpha/\gamma = 1$, then $R(t)$ and $Q(t)$ are s.v.f. and $P_t \sim [R(t)L_Q(t) + Q(t)L_R(t)]t^{-1}$.
- (iii) If $\theta \geq 1$, then $R(t)$ is a s.v.f. and $P_t \sim R(t)L_Q(t)t^{-\alpha/\gamma}$.
- (iv) If $\alpha/\gamma \geq 1$, then $Q(t)$ is a s.v.f. and $P_t \sim Q(t)L_R(t)t^{-\theta}$.

Theorem 3.2. Let $0 < \theta < 1$, $0 < \alpha/\gamma < 1$ and $t \rightarrow \infty$.

- (i) If $\theta + \alpha/\gamma > 1$, or $\theta + \alpha/\gamma = 1$ but $L_R(t)L_Q(t) \rightarrow 0$, then

$$P_t \sim tr(t)q(t)B(1 - \alpha/\gamma, 1 - \theta),$$

where $B(\cdot, \cdot)$ is Euler's beta function.

- (ii) If $\theta + \alpha/\gamma = 1$ but $L_R(t)L_Q(t) \rightarrow K \in (0, \infty)$, then $P_t \rightarrow P^* = 1 - e^{-K\pi/\sin\pi\theta}$.
- (iii) If $\theta + \alpha/\gamma < 1$, or $\theta + \alpha/\gamma = 1$ but $L_R(t)L_Q(t) \rightarrow \infty$, then $P_t \rightarrow 1$.

Comment 3.2. If $\theta + \alpha/\gamma > 1$, then

$$P_t \sim L_R(t)L_Q(t)B(1 - \alpha/\gamma, 1 - \theta)t^{-(\theta + \alpha/\gamma - 1)} \rightarrow 0.$$

If $\theta + \alpha/\gamma = 1$, then $P_t \sim L_R(t)L_Q(t)B(1 - \alpha/\gamma, 1 - \theta) \rightarrow 0$, i.e. the probability for non-hitting zero state develops like a s.v.f.

4. Limiting distributions. Note that $Z(t)$ has an invariant measure (see [14, Theorem 12.1]) whose p.g.f. is $U(s) = \int_0^s \frac{du}{f(u)-u}$, $0 \leq s \leq 1$. Then $U(F(s;t)) = U(s) + \mu t$ and $V(x) = U(1 - \frac{1}{x})$, $x \geq 1$, is increasing r.v.f. with an exponent γ . If $W(y)$ is the inverse function of $V(x)$, then it is also increasing r.v.f. with an exponent $1/\gamma$.

Introduce the increasing function $\Psi(x) = x^\alpha/l(x)$, $x \geq 1$. Then its inverse $\Psi(x)$, $x \geq 1$, is also increasing in $[1, \infty)$. Note that $g(s)$ can be presented in the following form

$$g(s) = 1 - \frac{1}{\Psi\left(\frac{1}{1-s}\right)}, \quad s \in [0, 1].$$

Let $\Delta(s) = \int_0^\infty q(u; s) du$, $s \in [0, 1]$. Then it is not difficult to prove that $\Delta(s) = \int_s^1 \frac{1-g(u)}{\mu(f(u)-u)} du$ and $Q = \Delta(0)$. Note that

$$\mathbf{E} \left[s^{Y(t)} | Y(t) > 0 \right] = 1 - (1 - \Phi(s; t)) / (1 - \Phi(0; t)).$$

Theorem 4.1. Let $\theta \geq 1$ such that $R = \int_0^\infty r(x) dx < \infty$, $\alpha/\gamma \geq 1$ such that $Q = \int_0^\infty q(x) dx < \infty$ and $t \rightarrow \infty$.

(i) If $q(t) = o(r(t))$, then

$$\mathbf{E}[s^{Y(t)} | Y(t) > 0] \rightarrow 1 - \frac{\Delta(s)}{Q} = \left(\int_0^s \frac{1-g(u)}{f(u)-u} du \right) / \left(\int_0^1 \frac{1-g(u)}{f(u)-u} du \right).$$

(ii) If $r(t) = o(q(t))$, then $\mathbf{P} \{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow D(\alpha, \gamma; x)$, $x \geq 0$, where

$$\hat{D}(\alpha, \gamma; \lambda) = \int_0^\infty e^{-\lambda x} dD(\alpha, \gamma; x) = 1 - \frac{\lambda^\alpha}{(1 + \lambda\gamma)^{\alpha/\gamma}}, \quad \lambda > 0.$$

(iii) If $r(t)/q(t) \rightarrow c \in (0, \infty)$, then

$$\mathbf{E}[s^{Y(t)} | Y(t) > 0] \rightarrow \frac{cQ}{cQ + R} \left(1 - \frac{\Delta(s)}{Q} \right)$$

and

$$\mathbf{P} \{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow \frac{cQ + RD(\alpha, \gamma; x)}{cQ + R}, \quad x \geq 0.$$

Comment 4.1. Since $q(t)/r(t) = t^{-(\alpha/\gamma - \theta)} L_Q(t)/L_R(t) \rightarrow 0$, then $\alpha/\gamma > \theta$ or $\alpha/\gamma = \theta$ but $L_Q(t)/L_R(t) \rightarrow 0$.

Consider a particular case when $L(s) \equiv 1/(1 + \gamma)$ and $l(s) \equiv 1$. Then for $\alpha > \gamma$ we obtain $\Delta(s) = \frac{1+\gamma}{\alpha-\gamma}(1-s)^{\alpha-\gamma}$ and $Q = \frac{1+\gamma}{\alpha-\gamma}$. Therefore, the limiting p.g.f. is $1 - \Delta(s)/Q = 1 - (1-s)^{\alpha-\gamma}$, which is the Sibuya distribution. For $\alpha = \gamma$ we have $\int_0^s \frac{1-g(x)}{f(x)-x} dx = (1+\gamma) \log \frac{1}{1-s}$ and therefore $Q = \infty$.

Theorem 4.2.

(i) If $\theta = 1$ such that $R(t) \rightarrow \infty$, $t \rightarrow \infty$, and $\alpha/\gamma > 1$, then

$$\mathbf{E}[s^{Y(t)}|Y(t) > 0] \rightarrow 1 - \Delta(s)/Q.$$

(ii) If $\theta > 1$ and $\alpha/\gamma = 1$ such that $Q(t) \rightarrow \infty$, $t \rightarrow \infty$, then

$$\mathbf{P}\{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow D(\alpha, \gamma; x), \quad x \geq 0,$$

where $D(\alpha, \gamma; x)$, $x \geq 0$ is defined in Theorem 4.1(ii).

Theorem 4.3. If $\theta = 1$ and $\alpha = \gamma$ such that $R(t) \uparrow R \leq \infty$, $Q(t) \uparrow \infty$, and $\frac{r(t)Q(t)}{q(t)R(t)} \rightarrow d \geq 0$, $t \rightarrow \infty$, then

$$\mathbf{P}\{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow D_{\gamma,d}(x), \quad x \geq 0,$$

where $D_{\gamma,d}(x)$ has a Laplace transform

$$\hat{D}_{\gamma,d}(\lambda) = \frac{d}{1+d} + \frac{1}{1+d} \cdot \frac{1}{1+\lambda^\gamma}, \quad \lambda > 0.$$

Comment 4.2. The conditional limiting distribution has an atom at zero with probability $\frac{d}{1+d}$. Then applying a Tauberian theorem one can obtain that $1 - D_{\gamma,d}(x) \sim x^{-\gamma}/[(1+d)\Gamma(1-\gamma)]$, i.e. the limiting r.v. belongs to a normal domain of attraction of a stable law with parameter γ . Note that $\alpha = \gamma$ and $\frac{L_R(t)Q(t)}{L_Q(t)R(t)} \rightarrow d$.

Theorem 4.4.

(i) If $\theta \geq 1$ and $0 < \alpha/\gamma < 1$, then

$$\mathbf{P}\{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow D(\alpha, \gamma; x), \quad x \geq 0,$$

where $D(\alpha, \gamma; x)$, $x \geq 0$ is defined in Theorem 4.1(ii).

(ii) If $0 < \theta < 1$ and $\alpha/\gamma \geq 1$ such that $Q = \int_0^\infty q(u) du < \infty$, then

$$\mathbf{E}\left[s^{Y(t)}|Y(t) > 0\right] \rightarrow 1 - \Delta(s)/Q.$$

Theorem 4.5. Let $0 < \theta < 1$, $0 < \alpha/\gamma < 1$, and $t \rightarrow \infty$.

(i) If $tr(t)q(t) \rightarrow 0$, then

$$\mathbf{P}\{Y(t)/W(\mu t) \leq x | Y(t) > 0\} \rightarrow D_1(\theta, \alpha, \gamma; x), \quad x \geq 0,$$

where $D_1(\theta, \alpha, \gamma; x)$ has a Laplace transform

$$\hat{D}_1(\theta, \alpha, \gamma; \lambda) = 1 - \frac{\lambda^\alpha}{B(1-\theta, 1-\frac{\alpha}{\gamma})} \int_0^1 (1-u)^{-\theta} (u\lambda^\gamma + 1)^{-\alpha/\gamma} du.$$

(ii) If $\text{tr}(t)q(t) \rightarrow K \in (0, \infty)$, then

$$\mathbf{P} \{Y(t)/W(t) \leq x | Y(t) > 0\} \rightarrow D_2(\theta, \alpha, \gamma; x), \quad x \geq 0,$$

where $D_2(\theta, \alpha, \gamma; x)$ has a Laplace transform

$$\hat{D}_2(\theta, \alpha, \gamma; \lambda) = 1 - \frac{1 - \exp \left(-K\lambda^\alpha \int_0^1 (1-u)^{-\theta} (u\lambda^\gamma + 1)^{1-\theta} du \right)}{1 - \exp(-K \cdot B(\theta, 1-\theta))}.$$

(iii) If $\text{tr}(t)q(t) \rightarrow \infty$, then

$$\mathbf{P} \left\{ Y(t)/\overleftarrow{\Psi}(W(t)) \leq x \right\} \rightarrow D_3(\alpha; x), \quad x \geq 0,$$

where $D_3(\theta, \alpha, \gamma; x)$ has a Laplace transform $\hat{D}_3(\alpha; \lambda) = e^{-\lambda^\alpha}$, $\lambda > 0$.

Comment 4.3.

(i) In this case $\text{tr}(t)q(t) = L_R(t)L_Q(t)t^{1-\theta-\alpha/\gamma} \rightarrow 0$, which means that $\theta + \alpha/\gamma < 1$ or $\theta + \alpha/\gamma = 1$ but $L_R(t)L_Q(t) \rightarrow 0$. The normalizing function is regularly varying with parameter $1/\gamma$ and we obtain a conditional limiting distribution $D_1(x)$. Then by the Tauberian theorem

$$1 - D_1(x) \sim x^{-\alpha} / [\Gamma(1-\alpha)(1-\theta)B(1-\theta, 1-\alpha/\gamma)], \quad x \rightarrow \infty,$$

i.e. the limiting r.v. belongs to a normal domain of attraction of a stable law with parameter α .

(ii) In this case $\text{tr}(t)q(t) = L_R(t)L_Q(t) \rightarrow K \in (0, \infty)$ because $\theta + \alpha/\gamma = 1$. Then, with the same normalizing function as in the previous case we obtain similarly that

$$1 - D_2(x) \sim Kx^{-\alpha} / \{\Gamma(1-\alpha)(1-\theta)[1 - \exp(-KB(\theta, 1-\theta))]\}, \quad x \rightarrow \infty.$$

(iii) In this case $\theta + \alpha/\gamma < 1$ or $\theta + \alpha/\gamma = 1$ but $L_R(t)L_Q(t) \rightarrow \infty$. The normalization is by a r.v.f. with parameter θ/α and surprisingly the non-conditional limiting distribution is just stable with parameter α .

5. Concluding remarks. As it was shown, the asymptotic behaviour of the non-visiting zero probability and the limiting distributions depend on the relations between the basic parameters: γ (reproduction), α (immigration), and θ (decreasing Poisson intensity), as well as on the corresponding s.v.f. (in some cases). Note that the intensity $r(t)$ can be interpreted as a control function because in the cases when $r(t)$ is increasing or converges to a positive constant the asymptotic behaviour of the processes is quite different and the obtained results are published in [1] and [2].

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