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INFLUENCE OF THE AUTO-UNLOADING SYSTEM ON THE ROTOR AXIAL OSCILLATIONS

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Abstract

This paper examines the rotor of a centrifugal pump with an automatic unloading system, which has free axial movement within the end clearance of a hydraulic balancer. The relationship between axial and radial hydrodynamic forces arising in the throttling gaps of the balancing device is demonstrated. Frequency responses of balancing systems are constructed, and an inequality defining the stability limit is obtained. It is noted that automatic balancing systems for the axial forces of a multistage centrifugal pump rotor simultaneously function as a self-adjusting non-contact seal and a loaded angular hydrostatic bearing, significantly determining the vibration state of the rotor.

Key words: auto-unloading system, axial vibrations, frequency characteristics, stability, hydrostatic support

Introduction. Today, one of the most effective methods of balancing axial forces for large-sized multistage high-pressure pumps is the use of automatic balancing devices that simultaneously perform the functions of a radial-face non-contact seal and a hydrostatic radial-thrust bearing [1]. This design allows for a significant reduction in the load on the axial supports, an increase in the reliability of the rotor, and a reduction in energy losses associated with friction and leaks of the working medium. Jin et al. [2] show how leaks, axial clearance, and hydraulic

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forces form the conditions of stability along the axial coordinate for an impeller with automatic axial self-balancing (auto-balanced impeller).

Despite the existence of various design solutions, all automatic balancing devices are built on a common principle consisting in creating negative feedback between the balancing force and the axial position of the rotor [3–5]. In a centrifugal pump, the rotor has a degree of freedom in the axial direction, moving within the final clearance of the so-called hydraulic heel. In the simplest case, it behaves as an absolutely rigid body performing one-dimensional axial oscillations, the frequency and amplitude of which are determined by the rigidity and damping of the balancing system [6]. This approach ensures stabilization of the rotor position with minimal deviations from the specified position, which is a key factor in the durability of seals and bearing assemblies [7]. This allows the system to effectively respond to changing loads in dynamic modes and maintain stability during pressure or flow fluctuations [8, 9].

Pavlenko et al. [10] propose a probabilistic method for calculating an automatic axial balancing device (ABD), considering geometric, hydromechanical and operational parameters as random variables.

Materials and methods. ***The model of the automatic balancing device.*** During the operation of the pump, disturbances close to harmonic in the form of discharge pressure pulsations and axial force act on the rotor. The pulsation frequency is equal to or a multiple of the rotation frequency. Under the influence of these perturbations, the rotor performs forced oscillations, the amplitude of which depends on the remoteness of the rotation frequency from the natural frequencies of the "rotor-balancing device" system. The scheme of the automatic balancing device model is shown in Fig. 1 [11].

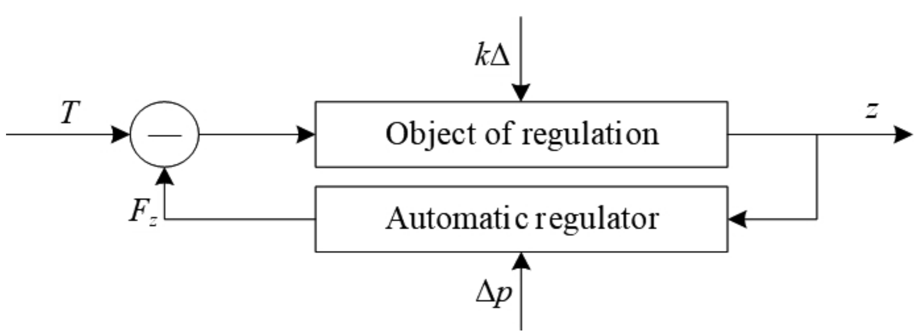

Fig. 1. Scheme of the balancing device model: z – end gap (adjustable); ΔP – sealing pressure; T – axial force; F_z – pressure force in the end gap; k, Δ – compression ratio and deformation of the pressing device

Axial vibrations of the rotor lead to the occurrence of significant stress pulsations in the unloading disk and in the shaft cross-section and can also cause increased transverse oscillations of the rotor. In this regard, the calculation of the amplitude and phase frequency characteristics of the balancing system and the verification of its dynamic stability are important for ensuring the reliability of high-speed high-pressure pumps.

Equations for the automatic unloading system dynamics. Consider the problem of obtaining linearized equations of joint radial-axial oscillations of the simplest single-mass model of a rigid rotor with an automatic balancing device in Fig. 2 [12].

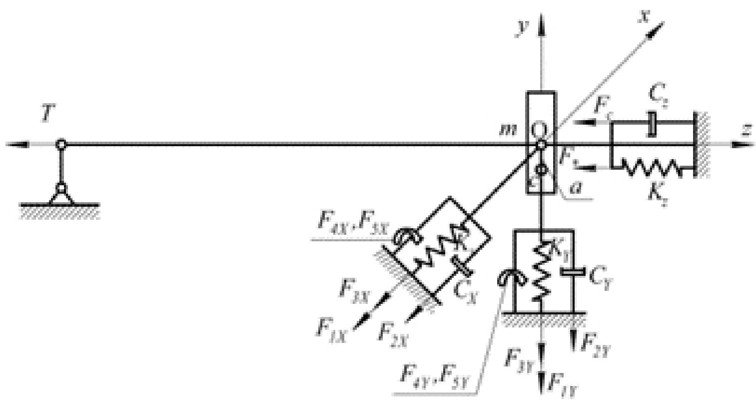

Fig. 2. Single-mass rigid rotor model with automatic balancing device

As equations of forced radial oscillations of a statically unbalanced rotor in projections on the axes of a fixed coordinate system, we use the equations [6]

$$
\begin{aligned}
&a_1\ddot{u}_x + a_2\dot{u}_x + a_3u_x + a_4\dot{u}_y + a_5u_y = \frac{a}{H_2}\omega^2\cos\omega t,\\
&a_1\ddot{u}_y + a_2\dot{u}_y + a_3u_y + a_4\dot{u}_x - a_5u_x = \frac{a}{H_2}\omega^2\sin\omega t,\\
&u_x = \frac{x}{H_2}, \quad u_y = \frac{y}{H_2},
\end{aligned} \tag{1}
$$

where H_2 is the base value of the end clearance of the hydraulic heel; ω is the frequency of own rotation of a single-mass rotor.

The dimensions of all terms in these equations are s^{-1}, and the coefficients for laminar flow regimes are determined by the formulas

$$
a_1 = 1 + \frac{k_g}{m}, \quad a_2 = \frac{1}{m}\left(k_d + k_gK_i\theta\right) = \frac{k_d}{m}\left(1 + \frac{p}{\mu}q_0\frac{H_1}{l_1}\theta\right), \tag{2}
$$

$$
a_3 = \frac{1}{m}k_p\left(\theta + 4\chi_s\right), \quad [s^{-2}], \quad a_4 = a_4'\omega, \quad a_4' = \frac{k_g\kappa}{2m}, \quad a_5 = a\_5\omega, \quad a_5' = \frac{k_d\kappa}{2m},
$$

where m is the reduced mass of the rotor; k_d, k_g, k_p are the coefficients of hydrodynamic forces in gap seals; K_i is the coefficient characterizing the influence of local acceleration on the damping radial force; κ is the coefficient of flow swirl in the annular gap; χ_s is the dimensionless axial force; ρ is the density of the liquid;

μ is the coefficient of dynamic viscosity; θ is the relative generalized angular coordinate; H_1 is the average radial clearance of the annular throttle; l_1 is the length of the ring throttle.

For turbulent flows, only the damping and hydrostatic stiffness coefficients change:

$$a_{2T} = \frac{1}{m}\left(k_{dt} + k_g K_{it}\theta\right) = \frac{k_{dt}}{m}\left(1 + 600\frac{H_1}{l_1}\theta\right), \quad a_{3T} = \frac{1}{m}k_p\left(\theta + \chi_{st}\right).$$

We multiply the second equation (1) by an imaginary unit, add both equations term by term and introduce a complex variable $u_r = u_x + iu_y$. As a result, instead of a fourth-order system, we get a compressed second-order system with complex coefficients

$$a_1\ddot{u}_r + a_2\dot{u}_r + a_3u_r - i\omega\left(a_4'\dot{u}_r + a_5'u_r\right) = \frac{a}{H_2}\omega^2 e^{i\omega t}$$

or in operator form

$$D_r(p)u_r = \omega^2\frac{a}{H_2}e^{i\omega t}, \tag{3}$$

where the eigenoperator of radial oscillations has the form

$$D_r(p) = a_1p^2 + a_2p + a_3 - i\omega\left(a_4'p + a_5'\right). \tag{4}$$

We linearize the nonlinear forces of viscous resistance and hydrostatic stiffness in the vicinity of static equilibrium, passing to their variations:

$$\delta\left(a_2\dot{u}_r\right) = a_{20}\delta\dot{u}_r + \dot{u}_{r0}\delta a_2 = a_{20}\delta\dot{u}_r \ (\dot{u}_{r0} = 0), \quad a_{20} = a_2 \ (\Delta p_1 = \Delta p_{10}),$$
$$\delta(a_3u_r) = a_{30}\delta u_r + u_{r0}\delta a_3, \quad a_{30} = a_3 \ (\Delta p_1 = \Delta p_{10}), \quad \delta a_3 = \frac{a_{30}}{\Delta p_{10}}(\delta p_1 - \delta p_2),$$

where u_{r0} is the given initial dimensionless radial deflection of the disk,

$$u_{r0} = \frac{\vec{r}_0}{H_2}, \quad \varepsilon_0 = \frac{|\vec{r}_0|}{H_1} = \overline{H} = \frac{H_2}{H_1},$$

$\vec{r}$ is the vector of radial displacement of the centre of the disk.

Variations in stiffness coefficients

$$\begin{aligned}\delta a_3 &= \delta\left[\frac{p_1 - p_2}{m}\cdot\frac{\pi R_1 l_1}{2H_1}(\Theta + 4\chi_s)\right] = \frac{a_{30}}{p_{10} - p_{20}}(\delta p_1 - \delta p_2)\\ &= \frac{a_{30}}{\sigma\Delta\psi_{10}}(\sigma\delta\psi_1 - \delta\varphi),\\ \delta a_{3\tau} &= \delta\left[\frac{p_1 - p_{2\tau}}{m}\cdot\frac{\pi R_1 l_1}{2H_1}(\Theta + \chi_{s\tau})\right] = \frac{a_{3\tau 0}}{\sigma\Delta\psi_{10}}(\sigma\delta\psi_1 - \delta\varphi_\tau)\end{aligned}$$

depend on the control action, which, in turn, depends on the axial position of the rotor. Here R_1 is the average radius of the annular gap, $\Delta\psi$ is the dimensionless pressure drop across the balancing device; ϕ is the dimensionless balancing force; σ is the dimensionless area;

$$\phi = \frac{F_z}{A_o P_n}, \quad \sigma = \frac{A_e}{A_o}.$$

Now equations (3) in variations (signs of variations are omitted) take the form

$$(5) \quad a_1 \ddot{u}_r + a_{20}\dot{u}_r + a_{30}u_r - i\omega\left(a_4'\dot{u}_r + a_5'u_r\right) + \frac{a_{30}u_{r0}}{\sigma\Delta\psi_{10}}\left(\sigma\psi_1 - \phi\right) = \omega^2\frac{a}{H_2}e^{i\omega t}.$$

We write equation (5) in operator form and use the expressions for the dimensionless balancing force for laminar and turbulent flows, relating radial and axial oscillations:

$$\phi = \kappa_s \frac{\tau_2 p + 1}{T_2 p + 1} u_z + \frac{k_1}{T_2 p + 1}\psi + \frac{k_2}{T_2 p + 1}\varepsilon,$$

where $u_z = z/H_2$ is the axial displacement referred to the base end clearance H_2; $\varepsilon = \frac{|\vec{r}|}{H_1} = \frac{|\vec{r}|}{H_2}\frac{H_2}{H_1} = |\vec{u}_r|\,|\vec{H}|$; T_2, τ_2 are the time constants; k_1, k_2 are the reduced shaft stiffness coefficients.

As a result, we get

$$\begin{aligned}\left[D_{r_0}(p)\left(T_2p+1\right) - k_2\overline{H}\beta_0 u_{r_0}\right]u_r - \beta_0 u_{r_0}\kappa_s(\tau_2 p+1)u_z \\ = \beta_0 u_{r_0}\left[k_1 - \sigma\left(T_2p+1\right)\right]\psi_{1a}e^{i\omega t} + \left(T_2p+1\right)\omega^2\frac{a}{H_2}e^{i\omega t},\end{aligned}$$

where β_0 is the cross coefficient of the angular rigidity of the shaft.

This equation, together with the equation for axial vibrations

$$D_z(p)u_z - \overline{H}Kk_2|u_r| = K[k_1\psi_{1a} - (T_2p+1)\tau_a]e^{i\omega t},$$

form a system of inhomogeneous differential equations with respect to generalized coordinates u_z and $u_r = u_x + iu_y = |u_r|e^{i\alpha}$; $|u_r| = \sqrt{u_x^2 + u_y^2}$

$$\begin{aligned}(6) \quad & D_z(p)u_z - \overline{H}Kk_2|u_r| = K[k_1\psi_{1a} - (T_2p+1)\tau_a]e^{i\omega t} \\ & - \beta_0 u_{r_0}\kappa_s(\tau_2p+1)u_z + [D_{r_0}(p)(T_2p+1) - k_2\overline{H}\beta_0 u_{r_0}]u_r \\ & = \beta_0 u_{r_0}[k_1 - \sigma(T_2p+1)]\psi_{1a}e^{i\omega t} + (T_2p+1)\omega^2\frac{a}{H_2}e^{i\omega t}.\end{aligned}$$

Based on formulas (2) $a_{30} = \Delta\psi_{10}$, therefore, the coefficient β does not depend on the discharge pressure. The system of equations (6) contains one real unknown u_z, and one complex unknown $u_r = u_x + iu_y = |u_r|e^{i\alpha}$.

Only the radial displacement modulus $|u_r|$ is of practical importance. On this basis, we accept $\alpha = 0$, $u_r = |u_r|$. Let us do the same with the initial offset vector $u_{r0} = |u_{r0}|e^{i\alpha_0}$, namely, write down $\alpha = 0$, $u_{r0} = |u_{r0}$. After such simplifications, equations (6) take the form

$$D_{zz}(p)u_z + d_{zr}|u_r| = \Phi_z, \quad D_{rz}(p)u_z + D_{rr}(p)|u_r| = \Phi_r. \tag{7}$$

The equations and their operators for the turbulent flow regime in throttling channels have a similar form. The operators d_{zr}, D_{rz} determine the coupling of axial and radial oscillations. Solving equations (7) for laminar and turbulent flows, we find

$$u_z = \frac{1}{D}(D_{rr}\Phi_z - d_{zr}\Phi_r), \quad |u_r| = \frac{1}{D}\left(D_{zz}\Phi_r - D_{rz}\Phi_z\right). \tag{8}$$

Own operator of the rotor-balancing device system, taking into account the coupling of axial and radial vibrations, has the form

(9)

$$D(p) = D_z(p)\left[D_{r_0}(p)\left(T_2p+1\right) - k_2\overline{H}\beta\right] - K\kappa_s k_2\overline{H}\beta\left(\tau_2p+1\right) = U_D - i\omega V_D.$$

It was proved in [9] that under the asymptotic stability of partial systems the coupled system is also asymptotically stable.

Let us group the external influences included on the right side of (8):

$$\begin{aligned} D(p)u_r &= \left(M_{r\psi}\psi_{1a} + M_{r\tau}\tau_a + M_{ra}\frac{a}{H_2}\right)e^{i\omega t} \\ D(p)u_z &= \left(M_{z\psi}\psi_{1a} + M_{z\tau}\tau_a + M_{za}\frac{a}{H_2}\right)e^{i\omega t}, \end{aligned} \tag{10}$$

where, taking into account (8), the operators of external actions have the form

$$M_{r\psi} = k_1\beta\left\{\left[1 - \frac{\sigma}{k_1}\left(T_2p+1\right)\right]D_z + K\kappa_s\left(\tau_2p+1\right)\right\}, \tag{11}$$

$$M_{r\tau} = -K\kappa_s\beta\left(\tau_2p+1\right)\left(T_2p+1\right), \quad M_{ra} = \omega^2\left(T_2p+1\right)D_z,$$

$$\begin{aligned} M_{z\psi} &= Kk_1\left(T_2p+1\right)\left(D_{r_0} - \frac{\sigma}{k_1}k_2\overline{H}\beta\right), \\ M_{z\tau} &= -K\left(T_2p+1\right)\left[\left(T_2p+1\right)D_{r_0} - \overline{H}k_2\beta\right], \\ M_{za} &= \overline{H}k_2\omega^2\left(T_2p+1\right). \end{aligned} \tag{12}$$

For turbulent flows, the structure of all obtained expressions remains unchanged, it is only necessary to add a subscript "t" to operators D_z, D_{r0}, time constants T_2, τ_2, and coefficients k_1, k_2, κ_s, β.

If in equations (10) we set the right parts equal to zero, then we obtain the equations of free oscillations. For a laminar flow (the expressions are similar for a turbulent flow), taking into account (9), we write

$$[U_D(p) - i\omega V_D(p)]\,|u_r| = 0, \quad [U_D(p) - i\omega V_D(p)]\,u_z = 0.$$

The general solution of such homogeneous equations has the form

$$|u_r| = u_{r\mathrm{acc}} e^{\lambda t}, \quad u_z = u_{z\mathrm{acc}} e^{\lambda t}. \tag{13}$$

Substituting (13) into equations (10), we obtain the characteristic equation

$$D(\lambda) = U_D(\lambda) - i\omega V_D(\lambda) = 0,$$

which can be represented in the form of two equations: $U_D(\lambda) = 0$, $V_D(\lambda) = 0$. The imaginary parts of the roots of these equations represent the natural frequencies of the system; the real parts characterize the change in time of the amplitudes of free oscillations. The right parts (external action) of equations (10) change according to the harmonic law with the rotor speed ω, therefore the reactions of the considered linear system are also harmonic functions with the same frequency

$$u_z = u_{za} e^{i(\omega t + \gamma_z)}, \quad u_r = u_{ra} e^{i(\omega t + \gamma_r)}, \tag{14}$$

where u_{za}, u_{ra} are the response amplitudes; γ_z, γ_r is a shift of phases of reactions relative to the phase ωt of external influence. For forced harmonic oscillations with a rotation frequency ω, the operator of differentiation with respect to time $p = i\omega$. Having performed such a replacement, we obtain new expressions for the real and imaginary parts of the eigenoperators (9):

$$D(i\omega) = U(\omega) + i\omega V(\omega), \tag{15}$$

$$\begin{aligned} U(\omega) &= -(m_0 - n_0)\omega^6 + (m_2 - n_2)\omega^4 - (m_4 - n_4)\omega^2 + m_6, \\ V(\omega) &= (m_1 - n_1)\omega^5 - (m_3 - n_3)\omega^3 + m_5 - n_5. \end{aligned} \tag{16}$$

Using coefficients (16), with the help of the modified Routh–Hurwitz criterion for polynomials with complex coefficients, it is possible to determine the stability of the oscillatory process by the signs of the real parts. The rotation frequencies at which eigenoperators (15) vanish are the eigenfrequencies of the system.

Results. ***Automatic unloading system frequency characteristics.*** The frequency transfer functions are equal to the ratios of responses to harmonic influences. To construct them in the action operators of equations (10), we perform a change and represent the operators (11) and (12) as complex numbers. As a result, we get

$$\begin{aligned} M_{r\psi} &= k_1\beta(U_{r\psi} + i\omega V_{r\psi}), & M_{z\psi} &= Kk_1(U_{z\psi} + i\omega V_{z\psi}), \\ M_{r\tau} &= -K\kappa_s\beta(U_{r\tau} + i\omega V_{r\tau}), & M_{z\tau} &= -K(U_{z\tau} + i\omega V_{z\tau}), \\ M_{ra} &= \omega^2(U_{ra} + i\omega V_{ra}), & M_{za} &= \overline{H}Kk_2\omega^2(U_{za} + i\omega V_{za}). \end{aligned} \tag{17}$$

Let us express the radial response to the harmonic change in discharge pressure from the first equation (10) using dependences (14), (15), and (17)

$$(U + i\omega V)_{r\psi}\, u_{r\psi}\, e^{i(\omega t + \gamma_{r\psi})} = (U_{r\psi} + i\omega V_{r\psi}) k_1 \beta \psi_{1a} e^{i\omega t}.$$

Then the corresponding frequency transfer function has the form

$$W_{r\psi}(i\omega) = \frac{u_{r\psi}}{\psi_{1a}} e^{i\gamma_{r\psi}} = A_{r\psi}(\omega) e^{i\gamma_{r\psi}(\omega)} = k_1 \beta \frac{U_{r\psi} + i\omega V_{r\psi}}{U + i\omega V}, \tag{18}$$

where $A_{r\psi}(\omega)$, $\gamma_{r\psi}(\omega)$ are the amplitude and phase frequency characteristics. To calculate them, we single out the real and imaginary parts of fraction (18). Multiplying the numerator and denominator by the complex number conjugate to the denominator, we get

$$W_{r\psi} = k_1 \beta \left(\frac{UU_{r\psi} + \omega^2 V V_{r\psi}}{U^2 + \omega^2 V^2} + i\omega \frac{UV_{r\psi} - VU_{r\psi}}{U^2 + \omega^2 V^2} \right).$$

The amplitude and phase of this complex number are

$$A_{r\psi}(\omega) = \frac{u_{r\psi}}{\psi_{1a}} = k_1 \beta \sqrt{\frac{U_{r\psi}^2 + \omega^2 V_{r\psi}^2}{U^2 + \omega^2 V^2}}, \quad \gamma_{r\psi}(\omega) = \arctan \omega \cdot \frac{UV_{r\psi} - VU_{r\psi}}{UU_{r\psi} + \omega^2 V V_{r\psi}}. \tag{19}$$

Similarly, the frequency characteristics are calculated from all external influences – discharge pressure, axial force, and static imbalance of the rotor.

Since, as dimensionless displacements $u_{ra} = r_a/H_2$, $u_{za} = z_a/H_2$, we obtain the formulas for the absolute values of the amplitudes

$$\begin{aligned} r_{r\psi} &= H_2 \psi_{1a} A_{r\psi}, & r_{r\tau} &= H_2 \tau_a A_{r\tau}, & r_{ra} &= a A_{ra}, \\ z_{z\psi} &= H_2 \psi_{1a} A_{z\psi}, & z_{z\tau} &= H_2 \tau_a A_{z\tau}, & z_{za} &= a A_{za}. \end{aligned}$$

The coefficients of the differential operators, with the help of which the amplitude and phase frequency characteristics are obtained, are calculated for the steady-state values of the radial e_0 and axial z_0 displacements of the disk centre. When the rotation frequency ω (disturbance frequency) coincides with one of the natural frequencies $\omega_j(\omega)$, the corresponding amplitude reaches its maximum value. Such speeds are critical. According to formulas (19) the amplitude and phase frequency characteristics of the balancing device are constructed as responses to harmonic external influences ψ_1 and τ_n at $\tau_n = 1$ ($T_n = 52.8$ kN) (Fig. 3). Curves 1, 2, and 3 represent the results obtained without taking into account inertia for relative pressures $\psi_1 = 0.63$; 1.0; 1.13 ($p_1 = 10$, 16, 18 MPa); curves 4, 5, 6 – at the same pressures, taking into account the inertia of the liquid. It can be seen from the amplitude frequency characteristics that the inertial resistance of the liquid in the throttling channels of the balancing devices has

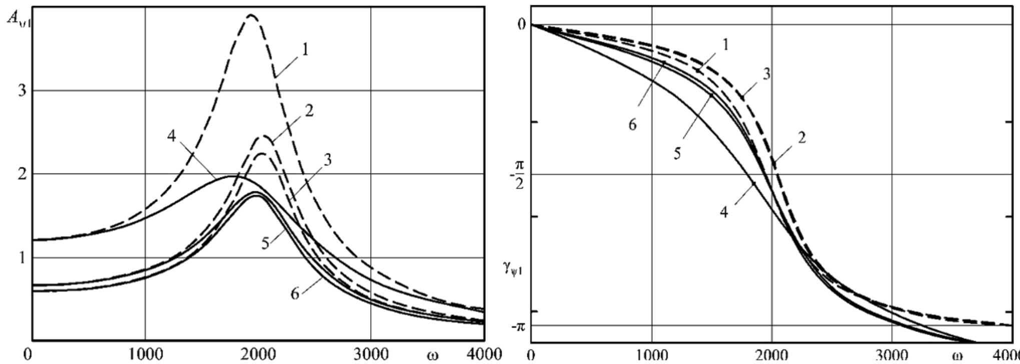

Fig. 3. Frequency characteristics for external influence, ψ_1, $\tau_n = 1$

a damping effect, reducing the natural frequencies and amplitudes of resonant oscillations. The effect of fluid inertia forces increases with decreasing discharge pressure. Therefore, taking this effect into account when predicting the dynamic characteristics of rotors is especially important for centrifugal pumps operating in a wide pressure range.

Stability of rotor axial vibrations. Using the algebraic Hurwitz stability criterion, one can estimate the stability of axial vibrations. For systems of the third order, neglecting the external damping ($\varsigma = 0$), we obtain the inequalities $\tau_2 > T_2$, $\tau_{2m} > T_{2m}$ [11]. After substituting the values of the time constants, these inequalities are reduced to the following form for laminar and turbulent regimes

$$V < \frac{A_e H E g_s^2 z_0}{3 Q_0^2}, \tag{20}$$

where A_e is the effective area of the hydro-heel disk; H is the nominal end clearance; Q_0, z_0 are the flow rate and clearance in steady state; g_s is the total conductivity of the radial and end chokes.

Inequality (20) limits the volume V of the hydro-heel chamber, at which the stability of the independent axial oscillations of the rotor is preserved.

Recent studies emphasize the role of mathematical modelling in analyzing physical processes in engineering systems. Lesko et al. in [12, 13] investigate sorption and neutralization of hazardous gases using algorithmic approaches to describe transport processes in multiphase media. Similar principles can be applied to the automatic rotor unloading system, where pressure distribution and fluid flow affect rotor axial stability. The influence of structural and thermomechanical factors on rotating equipment stability is also discussed in [14].

Conclusions. Automatic compensation systems for axial forces acting on the rotor of a multistage centrifugal pump significantly determine the rotor's oscillatory state. The automatic rotor unloading system, under the influence of radial static imbalance, outlet pressure pulsations, and harmonic changes in axial force, performs coupled radial–axial oscillations. At rotational speeds coinciding with natural frequencies, resonant amplitudes may exceed permissible limits; therefore, determining and avoiding resonant rotational frequencies is of great practical importance.

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