Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences

Tome 79, No 5, 2026

MATHEMATICS
Mathematical analysis

Corrigendum to "On spaceability of Besicovitch functions", [Comptes rendus de l'Académie Bulgare des Sciences 68(8) (2015), 957–966, DOI: 10.7546/CRABS.2019.06.07]

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The aim of this corrigendum is to fix or complete some parts of the article.

I. The part between formula (1) and the sentence "Let us denote by $\mathcal{B}$ ..." needs to be changed to

Let us fix a sequence $(x_n)_{n\geq 0}$ dense in $(0,1]$ and define the sequence $\boldsymbol{z} = (z_n)_{n\geq 0}$ by

$$z_0 = x_0,\ z_{4^\ell} = z_{4^\ell+1} = \cdots = z_{4^{\ell+1}-1} = x_\ell,\ \ell = 0,1,2,\ldots$$

II. The definition of the operator $T_z f$ (p. 958) should use g_{n-k} instead of g_n.

III. The original Theorem 1.2 should be replaced by

Theorem 1.2. *Let $\mathcal{B}$ and $T_{\boldsymbol{z}}$ be as above.*

(i) *For any nonzero $f \in \mathcal{B}$ the function $g = T_{\boldsymbol{z}}[f]$ is Besicovitch.*

(ii) *The operator $T_{\boldsymbol{z}}$ is linear, bounded and*

$$\overline{T_{\boldsymbol{z}}[\mathcal{B}]} \subseteq C([0,1])$$

is an infinite dimensional Banach space in which each function (but the zero function) is Besicovitch.

https://doi.org/10.7546/CRABS.2026.05.02

IV. The original Lemma 3.4 should be replaced by

Lemma 3.4. *Let* $g = \sum_{n=0}^{\infty} s_n g_n$ *be a function given by* (1) *such that* $(s_n)_n = o(n)$ *for* $n \to \infty$. *Let* $x \in [0,1]$ *be a point of infinite category with the corresponding nested sequence of L-segments* $(r^n_{m_n,p_n} = (\alpha_n, \beta_n))_{n=1}^{\infty}$. *If for some* $\delta \neq 0$ *and infinitely many k's,*

(12) (*originally* (2)) $$s_k = s_{k+1} = s_{k+2} = s_{k+3} = \cdots = s_{2k-1} = \delta,$$

then for each sufficiently large k fulfilling (12),

- $g(\alpha_k) = g(\beta_k) \leq g(x) \leq g((\alpha_k + \beta_k)/2)$ *if* $(-1)^k \delta > 0$ *and*
- $g((\alpha_k + \beta_k)/2) \leq g(x) \leq g(\alpha_k) = g(\beta_k)$ *if* $(-1)^k \delta < 0$.

Proof. We will assume that k is even and δ positive. The proof of the three remaining possibilities is analogous. We use Lemma 3.3 to express the function g at a point x as $g(x) = A_0^{k-1} + A_k^{k+3} + A_{k+4}^{\infty}$, where

$$A_i^j = \sum_{n=i}^{j} (-1)^n s_n \frac{2p_{n+1} - 1}{2^{m_1 + \cdots + m_{n+1}}}.$$

The construction of the functions g_n gives for each k, $g(\alpha_k) = g(\beta_k) = A_0^{k-1}$ and

$$g((\alpha_k + \beta_k)/2) = A_0^{k-1} + \frac{s_k}{2^{m_1 + \cdots + m_k}} = A_0^{k-1} + \frac{\delta}{2^{m_1 + \cdots + m_k}},$$

so it is sufficient to show that

(13) (*originally* (3)) $$0 \leq A_k^{k+3} - |A_{k+4}^{\infty}| \leq A_k^{k+3} + |A_{k+4}^{\infty}| \leq \frac{\delta}{2^{m_1 + \cdots + m_k}}.$$

We know that $m_n \geq 1$ and $1 \leq p_n \leq 2^{m_n - 1}$ for each n. It implies

(14) (*originally* (4)) $$\frac{1}{2^{m_n}} \leq \frac{2p_n - 1}{2^{m_n}} \leq 1 - \frac{1}{2^{m_n}}.$$

Using (12) and denoting $A = 1/2^{m_1 + \cdots + m_{k+3}}$ we can write

(*originally* (5)) $$|A_{k+4}^{\infty}| \leq 2A \left[\sum_{n=k+4}^{2k-1} \frac{\delta}{2^{m_{k+4} + \cdots + m_n + 1}} + \sum_{n=2k}^{\infty} \frac{|s_n|}{2^{m_{k+4} + \cdots + m_n + 1}} \right]$$
$$= 2A \left[\sum_{n=k+4}^{2k-1} \frac{\delta}{2^{m_{k+4} + \cdots + m_n + 1}} + \sum_{n=2k}^{\infty} \frac{|s_n|}{n} \frac{n}{2^{m_{k+4} + \cdots + m_n + 1}} \right]$$

$$\leq 2A\left[\frac{\delta}{2}+\sum_{n=2k}^{\infty}\frac{|s_n|}{n}\frac{n}{2^{n-k-2}}\right]$$
$$=2A\left[\frac{\delta}{2}+2^{k+2}\sum_{n=2k}^{\infty}\frac{|s_n|}{n}\frac{n}{2^n}\right]=(\spadesuit);$$

Since $(s_n)_n=o(n)$ for $n\to\infty$, we can choose k_0 so large that $\frac{|s_n|}{n}<\frac{\delta}{2}$ for each $k>k_0$ and $n\geq 2k$

$$(\spadesuit)\leq 2A\left[\frac{\delta}{2}+\frac{\delta}{2}2^{k+2}\sum_{n=2k}^{\infty}\frac{n}{2^n}\right]=2A\left[\frac{\delta}{2}+\frac{\delta}{2}\frac{2k+1}{2^{k-3}}\right]=(\clubsuit);$$

we can choose $k_1>k_0$ so large that $\frac{2k_1+1}{2^{k_1-3}}<1$. Then for each $k>k_1$ we obtain

$$(\clubsuit)=2A\left[\frac{\delta}{2}+\frac{\delta}{2}\frac{2k+1}{2^{k-3}}\right]<2A\delta=\frac{\delta}{2^{m_1+\cdots+m_{k+3}-1}}.$$

Summarizing, for each $k>k_1$ we have the inequality

$$B\cdot|A_{k+4}^{\infty}|\leq\frac{\delta}{2^{m_{k+1}+m_{k+2}+m_{k+3}-1}},$$

where we have put $B=2^{m_1+\cdots+m_k}$

Using our assumption on the values $s_k=s_{k+1}=s_{k+2}=s_{k+3}=\delta$ and (12), we get

$$B\cdot A_k^{k+3}\geq\frac{s_k-s_{k+1}}{2^{m_{k+1}}}+\frac{s_{k+1}}{2^{m_{k+1}+m_{k+2}}}+\frac{s_{k+2}-s_{k+3}}{2^{m_{k+1}+m_{k+2}+m_{k+3}}}+$$
$$+\frac{s_{k+3}}{2^{m_{k+1}+m_{k+2}+m_{k+3}+m_{k+4}}}=\frac{\delta}{2^{m_{k+1}+m_{k+2}}}+\frac{\delta}{2^{m_{k+1}+m_{k+2}+m_{k+3}+m_{k+4}}}$$

hence

$$BA_k^{k+3}-B|A_{k+4}^{\infty}|\geq$$
$$\geq\frac{\delta}{2^{m_{k+1}+m_{k+2}}}+\frac{\delta}{2^{m_{k+1}+m_{k+2}+m_{k+3}+m_{k+4}}}-\frac{\delta}{2^{m_{k+1}+m_{k+2}+m_{k+3}-1}}\geq 0.$$

This shows the leftmost inequality in (13). We have shown in the above inequalities that $BA_k^{k+3}\geq A_k^{k+3}\geq 0$. Using (12) we obtain after some standard computation

$$BA_k^{k+3}\leq\delta(1-\frac{1}{2^{m_{k+1}}})+\frac{\delta}{2^{m_{k+1}+m_{k+2}}}(1-\frac{1}{2^{m_{k+3}}})=$$
$$=\delta(1-\frac{1}{2^{m_{k+1}}}+\frac{1}{2^{m_{k+1}+m_{k+2}}}-\frac{1}{2^{m_{k+1}+m_{k+2}+m_{k+3}}})$$

hence

$$BA_k^{k+3} + B|A_{k+4}^{\infty}| \le \delta(1 - \frac{1}{2^{m_{k+1}}} + \frac{1}{2^{m_{k+1}+m_{k+2}}} - \frac{1}{2^{m_{k+1}+m_{k+2}+m_{k+3}}} + \\ + \frac{1}{2^{m_{k+1}+m_{k+2}+m_{k+3}-1}}) \le \delta$$

and the rightmost inequality in (13) follows. This proves the conclusion of the lemma. □

V. In the original Proposition 4.2:
the assumption $(s_n)_{n=0}^{\infty}$ *a nonzero admissible sequence* should be replaced by $(s_n)_{n\ge 0} = o(n)$ for $n \to \infty$, assume that for some $\delta \neq 0$ and infinitely many k's,

(*originally* (9, 10)) $$s_k = s_{k+1} = s_{k+2} = s_{k+3} = \cdots = s_{2k+1} = \delta.$$

and Lemma 3.4 applies.

VI. In the proof of Theorem 1.2(i):
If $f \in \mathcal{B}$ is nonzero, then the sequence $(s_n = f(z_n))_{n=0}^{\infty}$ satisfies the assumptions of both Propositions 4.1 and 4.2 hence $g = T_{\mathbf{z}}[f]$ is Besicovitch.

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